Physics 125c
Problem set number 1
Due Wednesday, April 7, 2004

Notes about course:

- There is no official text for this course. Instead, I will hand out various course notes on the topics we will cover. However, you may wish to refer to any of a large number of available books on quantum mechanics.
- Grade will be entirely on homework, that is, there will be homework sets as usual instead of a modterm or final.
- Homework will be handed out Wednesday, due the following Wednesday.
- Collaboration policy: OK to work together in small groups, and to help with each other's understanding. Best to first give problems a good try by yourself. Don't just copy someone else's work – whatever you turn in should be what you think you understand.
- There is a web page for this course, which should be referred to for the most up-to-date information. The URL:
  http://www.hep.caltech.edu/~fcp/ph125/
- TAs:
  - Anura Abeyesinghe, anura@caltech.edu;
  - Joan Erwin, rjerwin@caltech.edu;
  - Donal O'Connell, donal@caltech.edu.
- I have solutions to some of the problems on my web site. Do not look at these until after you have turned in the problem set!

READING: Read sections 1-4 of the "Approximate Methods" course note.

PROBLEMS:

1. Let us try an example of the discussion we have had in class concerning the use of the uncertainty relation on "localized" wave functions. Consider the three dimensional generalization. Hence, let $P(a)$ be the proability to find a particle of mass m in a sphere of radius a centered at the origin.

   (a) Recall that in the one dimensional case, if the probability of finding the particle in the interval $(-a, a)$ was α, then a simple lower bound on the kinetic energy was obtained as:

   $$T \geq \frac{1}{8m}\frac{\alpha^2}{a^2}. \tag{1}$$

   Make a simple, but rigorous, generalization of this result to the three dimensional case. Don't worry about finding the "best" bound; even a "conservative" bound may be good enough to answer some questions of interest.

   (b) We know that an atomic size is of order 10^{-10} m. Suppose that we have an electron which is known to be in a sphere of radius 10^{-10} m with 50% probability. What lower bound can you put on its kinetic energy? Is the result consistent with expectation; *e.g.*, with what you know about the kinetic energy of the electron in hydrogen?

   (c) In ancient times, before the neutron was discovered, it was supposed that the nucleus contained both electrons and protons. A comfortable nuclear size is 5×10^{-15} m. Find a lower bound on the kinetic energy of an electron if the probability to be within this radius is 90%. If there is a problem with the validity of your bound, see if you can fix it. Does this model encounter any difficulties?

   (d) Now find a lower bound for the kinetic energy of a proton in the nucleus, if it has a probability of 90% to be within a region of radius 5×10^{-15} m.

2. Do exercise 1 of the "Approximate Methods" course note.

3. Do exercise 2 of the "Approximate Methods" course note.

4. Do exercise 3 of the "Approximate Methods" course note.