

Physics 125c
 Problem set number 4 – Solution to Problem 13
 Due Wednesday, April 28, 2004

PROBLEMS:

13. In the notes we derived the optical theorem assuming that we had a “symmetric central force”. Show that this assumption is unnecessary. Hint: This is trivial, except for one piece of the assumption which you will have to retain.

Solution: Start with the step prior to making the assumption in the notes:

$$-\frac{i}{2\pi} \frac{\delta(p' - p'')}{p'} [f(\mathbf{p}', \mathbf{p}'') - f^*(\mathbf{p}'', \mathbf{p}')] = \frac{\delta(p' - p'')}{4\pi^2} \int_{(4\pi)} d\Omega_u f(\mathbf{p}', \mathbf{q}) f^*(\mathbf{p}'', \mathbf{q}). \quad (10)$$

Note that we must have $p' = p'' = q \equiv p$. Thus, write:

$$-\frac{i}{p} [f(p\mathbf{u}', p\mathbf{u}'') - f^*(p\mathbf{u}'', p\mathbf{u}')] = \frac{1}{2\pi} \int_{(4\pi)} d\Omega_u f(p\mathbf{u}', p\mathbf{u}) f^*(p\mathbf{u}'', p\mathbf{u}). \quad (11)$$

Now consider forward scattering: $\mathbf{u}'' = \mathbf{u}'$:

$$-\frac{i}{p} [f(p\mathbf{u}', p\mathbf{u}') - f^*(p\mathbf{u}', p\mathbf{u}')] = \frac{1}{2\pi} \int_{(4\pi)} d\Omega_u f(p\mathbf{u}', p\mathbf{u}) f^*(p\mathbf{u}', p\mathbf{u}). \quad (12)$$

With the assumption that $f(p\mathbf{u}', p\mathbf{u}) = f(p\mathbf{u}, p\mathbf{u}')$, we immediately see that we have once again the optical theorem:

$$\sigma_T(p) = \frac{4\pi}{p} \Im f(p; 1). \quad (13)$$

Note that the assumption we retained was that the scattering amplitude is invariant (up to a phase) under interchange of incoming and outgoing directions.

14. Do exercise 7 of the “Scattering” course note.
 15. Do exercise 8 of the “Scattering” course note.
 16. Do exercise 9 of the “Scattering” course note.