Physics 125c
Problem set number 6 – Solution to Problem
Due Wednesday, May 12, 2004

PROBLEMS:

20. Let's try an example application of our helicity formalism for angular distributions. Consider the magnetic dipole transition in which we radiatively excite a hydrogen atom from its ground ${}^1S_0$ state to the ${}^3S_1$ state, followed by subsequent decay back to the ground state via emission of a photon. That is, we consider the scattering process:

$$\gamma + {}^1S_0 \to {}^3S_1 \to \gamma + {}^1S_0. \tag{14}$$

This is an electromagnetic interaction, hence, parity is conserved. For unpolarized scattering, what angular distribution do you expect for the scattering angle between the initial and outgoing photon? Is your result valid for the laboratory frame, in which the initial hydrogen atom is at rest?

Solution: For unpolarize scattering, we must average over intial polarizations and sum over final polaraizations. The photon may have helicity states with $\lambda = \pm 1$, so we have:

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} \sum_{\lambda_i=-1,1} \sum_{\lambda_f=-1,1} \left|(2\cdot 1+1)T^1_{\lambda_i\lambda_f} d^1_{\lambda_i\lambda_f}(\theta)\right|^2. \tag{15}$$

The required little-d functions are:

$$d^1_{11}(\theta) = d^1_{11}(\theta) = \frac{1+\cos\theta}{2} \tag{16}$$

$$d^1_{1-1}(\theta) = d^1_{-11}(\theta) = \frac{1-\cos\theta}{2} \tag{17}$$

With parity conservation, plus an intermediate state that doesn't distinguish a photon coming from the left with positive helicity from a photon coming from the right with negative helicity, we can replace the four helicity amplitudes with one amplitude, T. Thus,

$$\frac{d\sigma}{d\Omega} = 3|T|^2\left[\left(\frac{1+\cos\theta}{2}\right)^2 + \left({}_2\frac{1-\cos\theta}{2}\right)^2\right] \tag{18}$$

$$= \frac{3}{2}|T|^2(1+\cos^2\theta). \tag{19}$$

We've done this calculation in the center-of-mass frame. However, this is an excellent approximation to the lab frame, in which the initial atom is at rest. To see this, compute the velocity, v, of the initial atom in the center-of-mass frame: Let p be the photon momentum (and energy) in the center-of-mass frame, M_0 be the rest mass of the initial (and final) atom, and M_1 be the rest mass of the intermediate excited atom. Then energy conservation gives, in the center-of-mass frame:

$$p + \sqrt{M_0^2 + p^2} = M_1. \tag{20}$$

But the energy levels of hydrogen are much closer together than the rest mass of hydrogen. To a good approximation then, $p + M_0 = M_1$. Thus, with $p = \gamma M_0 v \approx M_0 v$, we have

$$v \approx \frac{M_1 - M_0}{M_0} \ll O(10^{-9}). \tag{21}$$

21. Do exercise 1 of the "Identical Particles" course note.

22. Do exercise 2 of the "Identical Particles" course note.

23. Do exercise 3 of the "Identical Particles" course note.