

Physics 125c  
Problem set number 7 – Solution to Problem  
Due Wednesday, May 19, 2004

**PROBLEMS:**

24. We discussed the notion of the “Fermi sea” in class, according to which the energy levels occupied by the electrons in a metal are filled up under the restriction of the Pauli exclusion principle. In superconductivity, such as exhibited by niobium, there is a notion that zero resistance coherent motion can be achieved by pairing electrons into “Cooper pairs”. This is possible if there exists a long-range attractive force between two electrons in the metal. Knowing the critical temperature,  $T_c$ , below which superconductivity is achieved, make a crude estimate for how large a Cooper pair is. That is, make an estimate for the average distance between the two electrons of a Cooper pair. [Hints: The significance of the critical temperature is that it sets the scale of the attractive force, since if the temperature is large compared with the energy of attraction, the pairing will be broken by thermal excitations. Note that it is the electrons near the Fermi surface that are important, since they are the only ones where small energies could induce transitions among states. You may wish to attempt an argument based on the uncertainty principle.]

Make a numerical evaluation of your estimate for niobium. Roughly how many lattice spacings does your answer correspond to?

**Solution:** Let’s try a simple uncertainty principle argument for a starter: We know the temperature scale,  $T_c$  at which materials normally become superconducting. The typical interaction energy in a superconductor may be expected to be of this same scale. Thus, the uncertainty in the momentum is of order

$$\Delta k \approx \frac{T_c}{\varepsilon_F} k_F, \quad (22)$$

where  $k_F$  is the momentum at the Fermi surface, and  $\varepsilon_F = k_F^2/2m$ . Thus, we anticipate a typical spatial extent of order

$$\Delta x \approx \frac{1}{\Delta k} \approx \frac{\varepsilon_F}{T_c} \frac{1}{k_F}. \quad (23)$$

We need to estimate the energy of the Fermi surface. We obtained in class that the momentum is given by:

$$k_F = (3\pi^2 n)^{1/3}, \quad (24)$$

where  $n$  is the number density of conduction electrons. What is this density, numerically? Well, in a conductor, we have of order one electron contributed to the conduction band by each atom. Consider niobium, with atomic weight 93 and density 8.6 g/cc. We estimate the number density as:

$$n \approx \frac{8.6}{93} 6 \times 10^{23} \approx 6 \times 10^{22} \text{ electrons/cc} \quad (25)$$

Thus,

$$k_F \approx (180 \times 10^{22})^{1/3} \approx 1.2 \times 10^8 \text{ cm}^{-1} \quad (26)$$

In other units (using  $1 = 2 \times 10^{-5} \text{ eV-cm}$ , this is  $k_F \approx 2 \times 10^3 \text{ eV}$ . This corresponds to an energy of

$$\varepsilon_F \approx \frac{(2 \times 10^3)^2}{2 \times 0.5 \times 10^6} \approx 4 \text{ eV}. \quad (27)$$

This is the result we obtained in class. Now we proceed to estimate the Cooper pair “size”:

Let us take a value of  $T_c = 10 \text{ K}$  as a superconducting transition temperature for our estimate. This is approximately  $\frac{10}{300} \frac{1}{40} \approx 10^{-3} \text{ eV}$ . Thus,

$$\Delta x \approx \frac{1}{10^8} \frac{4}{10^{-3}} \approx 4 \times 10^{-5} \text{ cm}. \quad (28)$$

If the lattice spacing is of order  $10^{-8} \text{ cm}$ , then this distance corresponds to roughly 4000 lattice spacings.

25. Do exercise 1 of the “Electromagnetic Interactions” course note.
26. Do exercise 2 of the “Electromagnetic Interactions” course note.
27. Do exercise 3 of the “Electromagnetic Interactions” course note.