

Physics 129b
Problem Set Number 5
Due Wednesday, February 13, 2008

Reading: Read chapter 4 of Wu-Ki Tung, on irreducible vectors and operators. This short chapter parallels, and in some ways extends, the discussions in class and in the note on representation theory.

At last we are ready for a real physics application of group theory. We looked at the example of masses joined by springs in the shape of an equilateral triangle in class. Now, let us consider the problem of four masses joined by springs. The four masses are at the corners of a tetrahedron, and the springs form the edges of the tetrahedron. Thus, there are six springs connecting the four masses. All four masses are equal, and all six springs are identical.

We wish to determine the frequencies of the normal modes for this system. Notice that to solve the secular equation, $|V - \lambda I| = 0$, presents a formidable image. A little physical intuition can reduce it somewhat, but it would take real cleverness to solve it completely. This cleverness comes in the form of group theory! Group theory permits one to incorporate in a systematic and deliberate way everything we know about the symmetry of the problem, hence reducing it to a simpler problem.

The problem is still not trivial – you should spend time thinking about convenient approaches in setting things up, and about ways to avoid doing unnecessary work. Above all, be careful, and check your results as you proceed. The first three problems of this assignment take you the rest of the way through solving for the frequencies of the normal modes.

19. First step: Set up a 12-dimensional vector (coordinate system) describing the system, and derive the equations of motion, arriving finally at a set of linear equations that could be solved, in principle, to yield the frequencies of the normal modes.
20. Second step: Construct the character table for the tetrahedral symmetry group T_d .
21. Third step: Obtain the character table for the twelve-dimensional representation of the tetrahedral symmetry group that acts on your 12-dimensional vector describing the system. Decompose this representation into irreducible representations.
22. Final step: Obtain a small number of trace equations which you can use to solve to obtain the frequencies of the normal modes. Give the frequencies of the normal modes, and their degeneracies. Do your answers make physical sense?