Physics 135b
Problem set number 7
Due Wednesday, February 25, 2004

Reading: Read chapter 9 in the text, on quantum chromodynamics.

35. Let us get some more experience with calculating cross sections using Feynman graph perturbation theory. The text gives an example for $e^+e^- \to \gamma\gamma$ annihilation in the non-relativistic limit for the initial state. Now, we wish to consider the relativistic case. Hence, calculate the differential cross section for $e^+e^- \to \gamma\gamma$ in lowest non-zero order perturbation theory. Your goal is to obtain $\frac{d\sigma}{d\Omega}$, where Ω is the solid angle element around the direction of one of the outgoing photons. Do the calculation in the center-of-mass frame. Do the complete calculation, starting with the appropriate Feynman diagrams. Assume that the e^+ and e^- are unpolarized, and that we do not measure the photon polarizations. You may neglect the electron mass if you wish. Express your answer both in terms of the Mandelstam variables s, t, and u, and also in terms of $\cos\theta$, ϕ.

36. With reference to the preceding problem:

    (a) Calculate the total cross section for $e^+e^- \to \gamma\gamma$ to the same order. Be careful not to double count when doing this – remember that the two photons in the final state are indistinguishable. Do you have problems when performing the integral? If so, what can you do to fix it? Go ahead and fix it, if necessary, to a good approximation for high energies. Note that you should be able to do this without a lot of work, and to justify it. The answer is

$$\sigma_{e^+e^-\to\gamma\gamma} \approx \frac{2\pi\alpha^2}{s} \log\left(\frac{s}{m^2}\right),$$

    where m is the electron mass.

    (b) What is

$$R_{\gamma\gamma} \equiv \frac{\sigma_{e^+e^-\to\gamma\gamma}}{\sigma_{e^+e^-\to\mu^+\mu^-}}$$

    to this order in perturbation theory? Compute the numerical value of $R_{\gamma\gamma}$ at $E_{\rm cm} = m_Z$, according to quantum electrodynamics.

37. Problem 9.2 in text. Note that there is an error in the statement of the problem. I believe that Griffiths meant for you to work with $|3\rangle$ and $|8\rangle$, not $|7\rangle$ and $|8\rangle$.

38. Problem 9.3 in text.