Physics 135b
Problem set number 9
Due Wednesday, March 10, 2004

Reading: Read chapter 11 in the text, on gauge theories. This chapter summarizes some of the issues we have been discussing in class.

44. Problem 10.3 in text. If the answer is "no", draw a graph so that the comparison can be seen.

45. Problem 10.12 in text.

46. Problem 10.18 in text. Also, explain why ν_μ beams are easier to produce than ν_e beams at high energies.

47. Problem 11.21 in text.

48. We wrote down in class the Lagrangian for the electroweak interaction before spontaneous symmetry breaking and before mixing the neutral $SU(2)$ and $U(1)$ gauge bosons. In this problem, we wish to investigate the neutral gauge boson mass terms. The relevant terms in the Lagrangian are:

$$\mathcal{L} = \left| \left(-\frac{g}{2} \vec{\tau} \cdot \vec{W}_\mu - g' \frac{Y}{2} B_\mu \right) \phi \right|^2 ,$$

where it is convenient to represent the τ_i matrices by the Pauli matrices, and the "hypercharge" operator $Y = 2(Q - I_3^W)$.

(a) Carry out the symmetry breaking by substituting the vacuum choice

$$\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

in the above Lagrangian fragment, and obtain the terms involving the neutral gauge fields. Note that the electric charge of the Higgs boson is 0, since there is only one, and it therefore must be its own anti-particle. Your answer should depend on the constants g, g' and v, as well as the relevant fields.

(b) [Note: while the complete answer to this part depends on getting the answer to part a), most of this part can be accomplished without having done part a).] Now obtain the physical mass eigenstates. This involves carrying out the mixing of B_μ and W^3_μ to obtain mass eigenstates (in which the "mass matrix" in the Lagrangian must be diagonal):

$$\begin{aligned} A_\mu &= \cos\theta_W B_\mu + \sin\theta_W W^3_\mu \qquad \text{(photon)} \\ Z_\mu &= -\sin\theta_W B_\mu + \cos\theta_W W^3_\mu. \end{aligned} \tag{4}$$

Specifically, you are asked to obtain θ_W and m_Z in terms of g, g', and v. Also express m_Z in terms of m_W and θ_W, where m_W is the mass of the charged weak boson.