

Ph 135b: Solution Set 2

January 29, 2004

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a) Decay 1

$$\Xi^- \rightarrow \Lambda + \pi^+$$

corresponds to

$$dss \rightarrow uds + d\bar{u}$$

and so has one flavour changing vertex proportional to V_{us} whereas decay 2

$$\Xi^- \rightarrow n + \pi^-$$

corresponds to

$$dss \rightarrow udd + d\bar{u}$$

has two flavour changing vertices and so is far less probable. This corresponds to what is found experimentally with decay 1 occurring 99.88% and decay 2 $< 1.9 \times 10^{-5}$ of the time.

b) Decay 1

$$D^0 \rightarrow K^- + \pi^+$$

corresponds to

$$c\bar{u} \rightarrow s\bar{u} + u\bar{d}$$

which can go by (fig 1) and is proportional to $V_{cs} \cdot V_{ud}$.

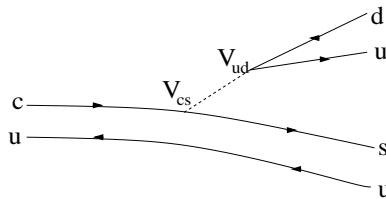


Figure 1:

Decay 2

$$D^0 \rightarrow K^+ + \pi^-$$

corresponds to

$$c\bar{u} \rightarrow u\bar{s} + d\bar{u}$$

which can go by (fig2) and which is proportional to $V_{cd} \cdot V_{us}$.

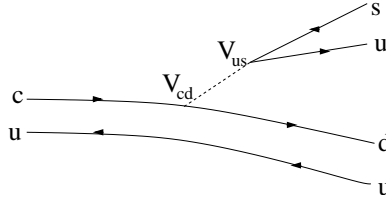


Figure 2:

Decay 3

$$D^0 \rightarrow \pi^- + \pi^+$$

corresponds to

$$c\bar{u} \rightarrow u\bar{d} + d\bar{u}$$

which can go by (fig 3) and is proportional to $V_{cd} \cdot V_{ud}$. Looking at the relative values of the CKM matrix

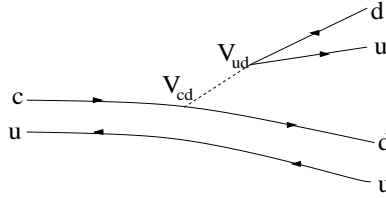


Figure 3:

elements we can see that decay 1 is the most likely and decay 2 the least likely. This corresponds to the experimental values; we have decay 1 3.8%, decay 2 $1.48 \times 10^{-4}\%$, decay 3 $1.43 \times 10^{-3}\%$ of the time. Looking at the relative strengths of the CKM matrix elements we can see that B mesons will go to D mesons and truthful mesons if they existed would go to B mesons.

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For the decay $M \rightarrow m_1 + \dots + m_n$ we want to maximise, say, p_1 with the constraints that $\sum p_i = 0$, $\sum E_i = M$ and $E_i = \sqrt{p_i^2 + m_i^2}$. With a little thought we see that this is a one dimensional problem. We can pick p_1 to be moving along the z-axis and all the other particles will be moving in the opposite direction. Now we want to maximise the function

$$F = p_1 + \lambda_1 \sum p_i + \lambda_2 (\sum E_i - M) + \sum \kappa_i (E_i^2 - p_i^2 - m_i^2)$$

We get the following equations

$$\frac{\partial F}{\partial p_i} = \delta_{i,1} + \lambda_1 - 2\kappa_i p_i = 0$$

$$\frac{\partial F}{\partial E_i} = \lambda_2 + 2\kappa_i p_i = 0$$

as well as the constraint equations. The first thing that we see is that for $i \neq 1$

$$\frac{p_i}{E_i} = \frac{-\lambda_1}{\lambda_2} = \text{constant}$$

i.e. all other particles are moving with the same velocity in the opposite direction. Now if we let $m_{tot} = \sum m_i$ we can write

$$p_1^2 = \left(\frac{M^2 - m_{tot}^2 + 2m_{tot}m_1}{2M} \right)^2 - m_1^2$$

this, considered as a function of m_1 , is maximised by setting $m_1 = m_{tot}/2$. So we see that we want one particle, the one with mass closest to $m_{tot}/2$, moving in one direction and all the others moving, with the same velocity as each other, in the opposite direction. In the example given we have $m(J/\psi) = 3096\text{MeV}$, $m(\Delta^{++}) = 1232\text{MeV}$, $m(\bar{p}) = 938\text{MeV}$, $m(\pi^-) = 140\text{MeV}$. So $m_1 = m(\Delta^{++})$ and hence

$$E(\Delta^{++}) = \frac{1}{2m(J/\psi)}(m(J/\psi)^2 + m(\Delta^{++})^2 - (m(\bar{p}) + m(\pi^-))^2) = 1605\text{MeV}$$

and $p(\Delta^{++}) = 1029\text{MeV}$.

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a) Unitary matrices, $U(N)$, satisfy $U^\dagger = U^{-1}$. Given $U_1, U_2 \in U(N)$ we have

$$(U_1 U_2)^\dagger = U_2^\dagger U_1^\dagger \quad (1)$$

$$= U_2^{-1} U_1^{-1} \quad (2)$$

$$= (U_1 U_2)^{-1}. \quad (3)$$

so the product $U_1 U_2$ is unitary and so we have closure. Obviously $I^\dagger = I = I^{-1}$ and so the identity matrix is unitary. If $U \in U(N)$ then $(U^{-1})^\dagger = (U^\dagger)^\dagger = U = (U^{-1})^{-1}$ and so U^{-1} is unitary. Finally from the definition of matrix multiplication we have associativity.

b) $SU(N)$ is the set of matrices which are unitary and have determinate 1. As $\det(AB) = \det(A)\det(B)$ and $\det(A^{-1}) = \det(A)^{-1}$ it is a subgroup of the unitary matrices and so is a group.

c) The proof that the set of orthogonal matrices, $O(N)$, form a group is the same as for unitary matrices but with the operation transpose instead of hermitian conjugate.

d) Using the same relations as in b) we can show that $SO(N)$ is a subgroup of $O(N)$.

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We have the following isospin assignments

Particle	Quarks	I	I ₃
Ω^-	sss	0	0
Σ^+	uus	1	1
Ξ^0	uss	1/2	1/2
ρ^+	$u\bar{d}$	1	1
η	$u\bar{u}$	0	0
\bar{K}^0	$\bar{d}s$	1/2	1/2
D^+	$c\bar{d}$	1/2	1/2
D^-	$\bar{c}d$	1/2	-1/2
D^0	$c\bar{u}$	1/2	-1/2
\bar{D}^0	$\bar{c}u$	1/2	1/2

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The particle Σ^0 has isospin state vector $|I = 1, I_3 = 0\rangle$. We can decompose this in the basis $|I_3)_1, (I_3)_2\rangle$ where we have labelled the state by the third isospin component of the two decay products. The result is

$$|I = 1, I_3 = 0\rangle = \frac{1}{\sqrt{2}}(|1, -1\rangle - |-1, 1\rangle)$$

so we would expect 50 decays into $\Sigma^+ + \pi^- = |1, -1\rangle$, 50 into $\Sigma^- + \pi^+ = |-1, 1\rangle$, and zero into $\Sigma^0 + \pi^0 = |0, 0\rangle$.

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a) η , being a pseudoscalar, has parity -1 and $J = 0$. Decay by strong or electromagnetic forces must conserve these. Photons have parity -1 as do the pions. The photon has spin 1 so the total orbital angular momentum must be 1 for $J = 0$. Hence the total parity

$$P_\pi^2 P_\gamma (-1)^l = 1$$

and so parity is not conserved ruling out this decay.

b) For the three pion decay parity is now conserved however G-Parity see pg 129-130 of Griffiths is not. η has G-Parity = $(C = 1)(R_2 = 1) = 1$ whereas three pions have G-parity of -1. This decay can only go by the electro-magnetic force. We can also note that if we examined the Feynman diagram which would give this decay by the strong force it would be suppressed by the OZI rule.