Ph 135b: Solution Set 4

February 12, 2004

18

a) We only have E_{CM} and G_F as parameters as we assume the masses involved are zero. The cross section has units of length squared or in natural units of energy^{-2}. The scattering amplitude is protortional to G_F and so $\sigma \propto G_F^2$. For correct units $\sigma \sim G_F^2 E_{CM}^2$.

b) We have the relation between c.o.m energy and the lab frame variables,

$$E_{CM}^2 = E^2 - p^2$$

where $E = E_\nu + M_p$ and as the neutrino is massless $E_\nu = p_\nu = p$. So

$$E_{CM}^2 \approx 2M_p E_\nu$$

for $E_\nu \gg M_p$. Thus

$$\begin{aligned} \sigma \sim 2G_F^2 E_\nu &= 2.7 \times 10^{-10} \text{fm}^2 && (1) \\ &= 2.7 \times 10^{-12} \text{barn}. && (2) \end{aligned}$$

19

$$\begin{aligned} s &= (p_a + p_b)^2 && (3) \\ t &= (p_a - p_c)^2 && (4) \\ u &= (p_a - p_d)^2 && (5) \end{aligned}$$

So that

$$\begin{aligned} s + t + u &= 3p_a^2 + p_b^2 + p_c^2 + p_d^2 + 2p_{a\cdot b} - 2p_{a\cdot c} - 2p_{a\cdot d} && (6) \\ &= 3m_a + m_b + m_c + m_d + 2p_a \cdot (p_b - p_c - p_d) && (7) \\ &= 3m_a + m_b + m_c + m_d - 2p_a \cdot (p_a) && (8) \\ &= m_a + m_b + m_c + m_d && (9) \end{aligned}$$

where we used $p_a + p_b = p_c + p_d$.

20

a)

$$N(t) = N(0) e^{-\Gamma t}$$

with $\Gamma = (2.2 \times 10^{-6})^{-1} s^{-1}$. As $t = 2.2 \times 10^{-5} s$ and $N(0) = 10 \times 10^6$ we have

$$N(t) = 45$$

b) The probability of pion lasting longer than 1 sec given by,

$$\frac{N(t=1s)}{N(0)} = e^{-\Gamma t}$$

which is negligible for $\Gamma = (2.6\times 10^{-8})^{-1}s^{-1}$ and $t = 1s$.

21

We use eqn. 6.34 from the text. As $\vec{p_2} = 0$,

$$(p_1\cdot p_2)^2 - (m_1 m_2)^2 = m_2^2|\vec{p_1}|.$$

We also have that $E_3 = |\vec{p_3}|$ and $E_4 = |\vec{p_4}|$. So,

$$\begin{aligned} d\sigma &= \frac{|\mathcal{M}|^2}{4}\frac{S}{|\vec{p_1}|m_2}\frac{d^3\vec{p_3}}{(2\pi)^3 2|\vec{p_3}|}\frac{d^3\vec{p_4}}{(2\pi)^3 2|\vec{p_4}|}(2\pi)^4\delta^4(p_1+p_2-p_3-p_4) && (10)\\ &= \left(\frac{|\mathcal{M}|}{8\pi}\right)^2\frac{S}{|\vec{p_1}||\vec{p_3}||\vec{p_4}|m_2}\delta(E_1+m_2-p_3-p_4)\delta^3(\vec{p_1}-\vec{p_3}-\vec{p_4})d^3\vec{p_3}d^3\vec{p_4} && (11)\end{aligned}$$

Integrating over $\delta^3(\vec{p_1}-\vec{p_3}-\vec{p_4})d^3\vec{p_4}$ we get,

$$d\sigma = \left(\frac{|\mathcal{M}|}{8\pi}\right)^2\frac{S}{|\vec{p_1}||\vec{p_3}|m_2}\frac{1}{E_4}\delta(E_1+m_2-p_3-E_4)d^3\vec{p_3} \qquad (12)$$

where $E_4 = |\vec{p_4}| = |\vec{p_1}-\vec{p_3}| = \sqrt{|\vec{p_1}|^2+|\vec{p_3}|^2-2|\vec{p_1}||\vec{p_3}|\cos\theta}$. To perform the integration over $|\vec{p_3}|$ we use the relation

$$\delta(f(x)) = \sum_i \frac{1}{\left|\left.\frac{df}{dx}\right|_{x_1}\right|}\delta(x-x_1)$$

with the sum over the zeros of $f(x)$. In our case, $x = |\vec{p_3}|$,

$$f(x) = E_1 + m_2 - x - \sqrt{|\vec{p_1}|^2 + x^2 - 2|\vec{p_1}|x\cos\theta}$$

with a zero at

$$x_1 = \frac{(E_1+m_2)^2 - |\vec{p_1}|^2}{2m_2 - 2E_1 - 2|\vec{p_1}|\cos\theta}$$

and

$$\frac{df}{dx} = -1 - \frac{(x-|\vec{p_1}|\cos\theta)}{\sqrt{|\vec{p_1}|^2+x^2-2|\vec{p_1}|x\cos\theta}} \qquad (13)$$

so

$$\left|\left.\frac{df}{dx}\right|_{x_1}\right| = \left.\frac{E_4 + x - |\vec{p_1}|\cos\theta}{E_4}\right|_{x_1} \qquad (14)$$

using $E_1 + m_2 - x_1 - E_4|_{x_1} = 0$ we get

$$d\sigma = \left(\frac{|\mathcal{M}|}{8\pi}\right)^2\frac{S}{|\vec{p_1}|m_2}\frac{|\vec{p_3}|}{(E_1+m_2-|\vec{p_1}|\cos\theta)}d\Omega \qquad (15)$$

as required.

18

a) No, this process is not allowed. It is impossible to draw a diagram that corresponds to this process. See part b).

b) We note that at each vertex one of each A,B and C particles meet. Consider a general diagram with E_A external A particles (similarly for B and C) and I_A internal A propagators. We must have $E_A + 2I_A$ vertices and similarly for B and C. Hence we see that $E_A = E_B = E_C | \, mod(2)$. The process in part a) does not satisfy this requirement i.e. $E_A = 1 mod 2$ but $E_B = 0 mod 2$.

c) The next order decays are $A \to 3C + B$ and $A \to 3B + C$ with the most likely being determined by the relative masses of B and C. The diagrams are shown below.

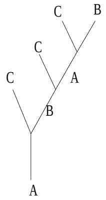

Figure 1: Process 1

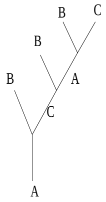

Figure 2: Process 2

19

In the formula for, $\frac{d\Gamma}{d\Omega}$ the angular dependence is entirely given by the matrix element. If we assume the interaction Hamiltonian is rotationally invariant we have that

$$\frac{d\Gamma}{d\Omega} \propto |\Psi(\theta, \phi)|^2$$

where, $\Psi(\theta, \phi)$, the probability of finding a particle in a infinitesimal solid angle $d\Omega$ at (θ, ϕ), is given by $Y_{1,1}(\theta, \phi) \propto \sin\theta$ for a state with spatial angular momentum $L = 1, L_z = 1$. Hence

$$\frac{d\Gamma}{d\Omega} \propto \sin^2\theta$$

as the ϕ dependence drops out when we take the norm.