Ph 135b: Solution Set 8

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We want to modify equation (7.165) to get $\Gamma(\eta_c \to 2\gamma)$. There are two considerations i) α includes a factor of e^2 which must be replaced with $Q^2e^2 = 4/9e^2$ to correct for the quark charge. ii) The in-states now carry a color factor so our amplitude has a factor of $\bar{c}_2 c_1$. If the quarks are in a singlet state

$$\frac{1}{\sqrt{3}}(\bar{r}r + \bar{b}b + \bar{g}g)$$

we get a factor of $\sqrt{3}$ in the amplitude. So we have

$$\begin{aligned}\frac{\Gamma(\eta_c \to 2g)}{\Gamma(\eta_c \to 2\gamma)} &= \frac{8\alpha_s^2}{3}\left(4 \times 3\left(\frac{4\alpha}{9}\right)\right)^{-1} \\ &= \frac{9}{8}\left(\frac{\alpha_s}{\alpha}\right)^2.\end{aligned}$$

If we take $\alpha_s = 0.2$ (see pg 165) we get a numerical result of 840. The experimental value is 830.

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For physicist A,

$$\alpha_s(|q|^2) = \frac{\alpha_s(\mu_A^2)}{1 + \left(\frac{\alpha_s(\mu_A^2)}{12\pi}\right)(11n - 2f)\ln\left(\frac{|q|^2}{\mu_A}\right)}$$

which implies that,

$$\alpha_s(\mu_B^2) = \frac{\alpha_s(\mu_A^2)}{1 + \left(\frac{\alpha_s(\mu_A^2)}{12\pi}\right)(11n - 2f)\ln\left(\frac{\mu_B}{\mu_A}\right)}$$

so we can write

$$\begin{aligned}\alpha_s(\mu_A^2) &= \alpha_s(\mu_B^2)\left(1 + \left(\frac{\alpha_s(\mu_A^2)}{12\pi}\right)(11n - 2f)\ln\left(\frac{\mu_B}{\mu_A}\right)\right) \\ \Rightarrow \alpha_s(\mu_A^2) &= \frac{\alpha_s(\mu_B^2)}{1 - \left(\frac{\alpha_s(\mu_B^2)}{12\pi}\right)(11n - 2f)\ln\left(\frac{\mu_B}{\mu_A}\right)}\end{aligned}$$

so we have,

$$\begin{aligned}\alpha_s(|q|^2) &= \left[\frac{\alpha_s(\mu_B^2)}{1-\left(\frac{\alpha_s(\mu_B^2)}{12\pi}\right)(11n-2f)\ln\left(\frac{\mu_B}{\mu_A}\right)}\right] \\ &\times \left[\frac{1}{1+\left(\frac{\alpha_s(\mu_B^2)/12\pi}{1-\left(\frac{\alpha_s(\mu_B^2)}{12\pi}\right)(11n-2f)\ln\left(\frac{\mu_B}{\mu_A}\right)}\right)(11n-2f)\ln\left(\frac{|q|^2}{\mu_A}\right)}\right] \\ &= \frac{\alpha_s(\mu_B^2)}{1-\left(\frac{\alpha_s(\mu_B^2)}{12\pi}\right)(11n-2f)\ln\left(\frac{\mu_B}{\mu_A}\right)+\left(\frac{\alpha_s(\mu_B^2)}{12\pi}\right)(11n-2f)\ln\left(\frac{|q|^2}{\mu_A}\right)} \\ &= \frac{\alpha_s(\mu_B^2)}{1+\left(\frac{\alpha_s(\mu_B^2)}{12\pi}\right)(11n-2f)\ln\left(\frac{|q|^2}{\mu_B}\right)}.\end{aligned}$$

as required. Now,

$$\ln\Lambda = \ln m_Z^2 - \frac{12\pi}{(11n-2f)\alpha_s(m_Z^2)}$$

where $m_Z = 92,600 \pm 2MeV$, $\alpha_s(m_Z^2) = 0.1172 \pm .0020$ and we have $f = 5$ since the top quark is not counted as it is too massive to be formed in a loop. Note that the error in $\alpha_s(m_Z^2)$ is much larger than that in m_Z. Using these expressions we get that $\Lambda = 85MeV$ and using

$$\begin{aligned}\Delta(\Lambda)^2 &= \left(\frac{\partial\Lambda}{\partial\alpha_s}\right)^2 \Delta\alpha_s^2 \\ &\sim 10MeV\end{aligned}$$

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a)Replacing the weak vertex with,

$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1+\epsilon\gamma^5)$$

we get the amplitude,

$$\mathcal{M} = \frac{g_w^2}{8(M_W c)^2}[\bar{u}(3)\gamma^\mu(1+\epsilon\gamma^5)u(1)][\bar{u}(4)\gamma^\mu(1+\epsilon\gamma^5)u(2)]$$

so that

$$\begin{aligned}\sum|\mathcal{M}|^2 &= \left(\frac{g_w^2}{8(M_W c)^2}\right)^2 Tr[\gamma^\mu(1+\epsilon\gamma^5)(\not{p}_1+m_e)\gamma^\nu(1+\epsilon\gamma^5)\not{p}_3] \\ &\times Tr[\gamma_\mu(1+\epsilon\gamma^5)\not{p}_2\gamma_\nu(1+\epsilon\gamma^5)(\not{p}_4+m_\mu)]\end{aligned}$$

using the usual trace identities we get,

$$\sum |\mathcal{M}|^2 = \left(\frac{g_w^2}{8(M_W c)^2}\right)^2 [4(1+\epsilon^2)(p_1^\mu p_3^\nu + p_1^\nu p_3^\mu - g^{\mu\nu} p_1\cdot p_3) + 8\epsilon i \epsilon^{\mu\nu\alpha\beta}(p_1)_\alpha (p_3)_\beta]$$
$$\times[4(1+\epsilon^2)((p_2)_\mu (p_4)_\nu + (p_2)_\nu (p_4)_\mu - g_{\mu\nu} p_2\cdot p_4) + 8\epsilon i \epsilon_{\mu\nu\gamma\delta} p_2^\gamma p_4^\delta]$$
$$= \left(\frac{g_w^2}{8(M_W c)^2}\right)^2 [16(1+\epsilon^2)^2(2p_1\cdot p_2 p_3\cdot p_4 + 2p_1\cdot p_4 p_2\cdot p_3)$$
$$+ \quad 128\epsilon^2(\delta_\delta^\alpha \delta_\gamma^\beta - \delta_\gamma^\alpha \delta_\delta^\beta) p_1^\alpha p_2^\delta p_3^\beta p_4^\gamma]$$
$$= \frac{1}{2}\left(\frac{g_w^2}{(M_W c)^2}\right)[(1+6\epsilon^2+\epsilon^4)(p_1\cdot p_2)(p_3\cdot p_4) + (1-\epsilon^2)^2(p_1\cdot p_4)(p_2\cdot p_3)]$$

b)Setting $m_e = m_\mu = 0$ and in the c.o.m. frame,

$$p_1 = (E,0,0,E), \qquad p_2 = (E,0,0,-E),$$
$$p_3 = (E,0,-E\sin\theta,-E\cos\theta) \qquad p_4 = (E,0,-E\sin\theta,-E\cos\theta)$$

Now,

$$< |\mathcal{M}|^2 >= \left(\frac{g_w^2 E^2}{2(M_W c)^2}\right)^2 [4(1+6\epsilon^2+\epsilon^4) + (1-\epsilon^2)^2(1-\cos\theta)^2]$$

So,

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{g_w^4 E^2}{16(M_W c)^4}[4(1+6\epsilon^2+\epsilon^4) + (1-\epsilon^2)^2(1-\cos\theta)^2]$$

and integrating over ϕ and θ using $2\pi\int_{-1}^{1}(1+\cos\theta)^2 d(\cos\theta) = 16\pi/3$ we get for the cross section,

$$\sigma = \pi\left(\frac{\hbar c}{8\pi}\right)^2 \frac{g_w^4 E^2}{(M_W c)^4}[(1+6\epsilon^2+\epsilon^4) + 1/3(1-\epsilon^2)^2]$$

c) The difference between the forward and backward scattering cross sections is propotional to $(1-\epsilon^2)^2$ and so can be used to determine its value.

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Tauon decay is the very same as the muon decay in eqn. 10.37 but with m_μ replaced with m_τ. However as well as decaying into a $e, \bar{\nu}_e, \nu_\tau$ it can decay into $\mu, \bar{\nu}_\mu, \nu_\tau$ and ρ, ν_τ. If we assume that m_μ, m_u, m_d, m_s can all be neglected in comparison with m_τ then the life times are exactly the same. We also note that we have three colors so that we have in effect 5 decay channels all with the same lifetime. Hence $\tau_\tau = \frac{1}{5}\left(\frac{m_\mu}{m_\tau}\right)^5 \tau_\mu$ which has a value of $3\times10^{-13}s$ compared with the experimental result $2.9\times10^{-13}s$.

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We wish to show the invariance of $1/4(G_{\mu\nu}^a (G^a)^{\mu\nu})$ where

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g f^{abc} G_\mu^b G_\nu^c$$
$$G_\nu^a \to G_\nu^a - 1/g\partial_\nu \alpha^a - f^{abc}\alpha^b G_\nu^c$$

so to first order in α

$$\begin{aligned}G^a_{\mu\nu} \quad \rightarrow \quad & \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g f^{abc} G^b_\mu G^c_\nu \\ & -1/g(\partial_\mu \partial_\nu \alpha^a - \partial_\nu \partial_\mu \alpha^a) \\ & +f^{abc}(\partial_\mu \alpha^b G^c_\nu + \partial_\nu \alpha^c G^b_\mu) \\ & -f^{abc}(\partial_\mu(\alpha^b G^c_\nu) - \partial_\nu(\alpha^b G^c_\mu)) \\ & +g f^{abc}(f^{bde}\alpha^d G^e_\mu G^c_\nu + f^{cde}\alpha^d G^e_\nu G^b_\mu)\end{aligned}$$

Simplifying this equation and using the Jacobi Identity

$$f^{ade}f^{bcd} + f^{cde}f^{abd} + f^{bde}f^{cad} = 0$$

we get

$$\begin{aligned}G^a_{\mu\nu} \quad & \rightarrow \quad G^a_{\mu\nu} - f^{abc}\alpha^b(\partial_\mu G^c_\nu) - \partial_\nu G^c_\mu)) + g f^{abc}\alpha^b(f^{dec}G^d_\mu G^e_\nu) \\ & = \quad G^a_{\mu\nu} - f^{abc}\alpha^b G^c_{\mu\nu}.\end{aligned}$$

Putting this into the Lagrangian

$$\delta\mathcal{L} = 1/4(-f^{abc}\alpha^b G^c_{\mu\nu}(G^a)^{\mu\nu} - G^a_{\mu\nu} f^{abc}\alpha^b (G^c)^{\mu\nu})$$

which is clearly zero as symmetric indicies are contracted with antisymmetric indicies.