Ph 135b: Solution Set 9

March 18, 2004

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a)Replacing the weak vertex with,

$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu$$

we get the amplitude,

$$\mathcal{M} = \frac{g_w^2}{8(M_W)^2}[\bar{u}(3)\gamma^\mu u(1)][\bar{u}(4)\gamma_\mu v(2)]$$

so that

$$\sum |\mathcal{M}|^2 \quad = \quad \frac{1}{2}\left(\frac{g_w^2}{(M_W)^2}\right)[(p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)].$$

This result can be found from last weeks solution by setting $\epsilon = 0$. For $m_e = 0$ and $p_1 = (m_\mu, 0, 0, 0)$ we have the following expressions,

$$p_1 \cdot p_2 = m_\mu E_2 \qquad p_1 \cdot p_4 = m_\mu E_4$$
$$p_2 \cdot p_3 = 1/2(m_\mu^2 - 2m_\mu E_4) \qquad p_4 \cdot p_3 = 1/2(m_\mu^2 - 2m_\mu E_2)$$

Now,

$$< |\mathcal{M}|^2 >= \left(\frac{g_w^2}{2(M_W)^2}\right)^2 [1/2m_\mu^2 E_4(m_\mu^2 - 2E_4) + 1/2m_\mu^2 E_2(m_\mu^2 - 2E_2)]$$

So with the analysis of pg 305-306 we get

$$d\Gamma = \left(\frac{1}{8}\right)^2 \frac{g_w^4 m_\mu}{64\pi^4 M_W^4}\frac{d^3\vec{p}_4}{E_4^2}\int_{1/2m_\mu - E_4}^{1/2m_\mu}[m_\mu^2 E_4(m_\mu^2 - 2E_4) + m_\mu^2 E_2(m_\mu^2 - 2E_2)]dE_2$$

,integrating and setting $E_4 = E$,

$$\frac{d\Gamma}{dE} = \left(\frac{\pi}{2}\right)\frac{g_w^4}{(4\pi M_W)^4}m_\mu E^2[3/2m_\mu - 8/3E]$$

This is obviously different than equation 10.35. We plot both equations below for $0 < E < 1/2m_\mu$. Note that in the plot energy is measured in units of m_μ and the spectra are normalised.

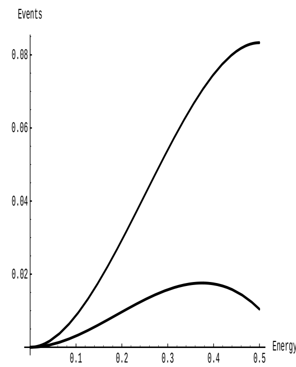

Figure 1: Normalised spectra

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We want to calculate

$$\begin{aligned}\frac{\Gamma(K^- \to e^- + \bar{\nu}_e)}{\Gamma(K^- \to \mu^- + \bar{\nu}_\mu)} &= \frac{m_e^2(m_K^2 - m_e^2)^2}{m_\mu^2(m_K^2 - m_\mu^2)^2} \\ &= 2.57 \times 10^{-5}.\end{aligned}$$

This compares with the experimental value of 2.44×10^{-5}. Now we are given that $\Gamma^{-1} = 1.2 \times 10^{-8}s$ and that $\Gamma_\mu = 0.64\Gamma$ and from,

$$\Gamma_\mu = \frac{f_k^2}{\pi\hbar m_k^3}\left(\frac{g_w}{4M_W}\right)^4 m_\mu^2(m_k^2 - m_\mu^2)$$

we can determine $f_k = 36MeV$.

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If we want to calculate, say $e^- + \nu_e \to e^- + \nu_e$, there are two Feynman diagrams that contribute to this process. It can be mediated by either a charged or a neutral current as opposed to only neutral currents in the $\mu^- + \nu_\mu \to \mu^- + \nu_\mu$ process.

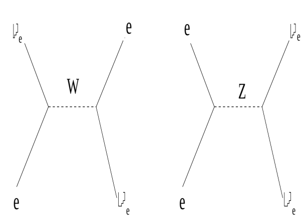

Figure 2: Feynman Diagrams for $e^- + \nu_e \to e^- + \nu_e$

Muon neutrinos are easier to experimentally produces because pions, which are copiously produced in proton colliders, preferentially decay into muons and muon neutrinos. See the previous problem.

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The interaction Lagrangian is, $\mathcal{L}_{int} = \alpha_Y \bar{\psi}\psi\phi$ and the free part is $\mathcal{L}_{free} = \mathcal{L} - \mathcal{L}_{int}$. Looking at the Euler-Lagrange equations for the free Lagrangian we see that ψ is a spin 1/2 particle of mass m_1 and so has a propagator of $\frac{i}{\not{p} - m_1 c}$. Similarly ϕ is a spin-0 particle with mass m_2 and has a propagator $\frac{i}{p^2-(m_2c)^2}$. The vertex is drawn below and carries a factor of $i\alpha_Y$. The solid ,arrowed lines are the fermions and the dashed line the scalar.

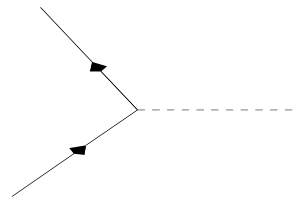

Figure 3: Vertex

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a)We are given a Lagrangian

$$\mathcal{L} = \left| \left(\frac{-g}{2} \vec{\tau} \cdot \vec{W}_\mu - \frac{g'}{2} Y B_\mu \right) \phi \right|^2$$

where $\vec{\tau}_i = \vec{\sigma}_i$. Now we expand about

$$\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

.

We are only interested in the neutral part so,

$$\begin{aligned} \mathcal{L} &\sim \left| \left(\frac{-g}{2} \begin{pmatrix} W_\mu^3 & 0 \\ 0 & -W_\mu^3 \end{pmatrix} - \frac{g'}{2} \begin{pmatrix} B_\mu & 0 \\ 0 & B_\mu \end{pmatrix} \right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2 \\ &= \frac{v^2}{8} \left(-gW_\mu^3 + g'B_\mu \right)^2 \end{aligned}$$

b) We define,

$$\begin{pmatrix} Z^0 \\ A \end{pmatrix} = \begin{pmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} W^3 \\ B \end{pmatrix}$$

where we have dropped the Lorentz indicies on W^3, B etc. Looking at the Lagrangian fragment we see that we have a mass term for a neutral boson. So from

$$Z^0 = \frac{1}{\sqrt{g^2 + g'^2}} (-gW_3 + g'B)$$

with mass, $M_Z = v/2\sqrt{g^2 + g'^2}$, we have, $\cos\theta_w = \frac{-g}{\sqrt{g^2+g'^2}}$ and $\sin\theta_w = \frac{-g'}{\sqrt{g^2+g'^2}}$, or $\tan\theta_w = g'/g$. A_μ the boson orthogonal to Z_0 is massless and proportional to $(gW_3 + g'B)$. If we had kept the charged part of the Lagrangian we would have seen that there are two bosons of mass $gv/2$ hence we get that $M_W = \cos\theta_W M_Z$ as required.