Physics 195a
Problem set number 4
Due 2 PM, Thursday, October 31, 2002

Notes about course:

- Homework should be turned in to the TA's mail slot on the first floor of East Bridge.
- Collaboration policy: OK to work together in small groups, and to help with each other's understanding. Best to first give problems a good try by yourself. Don't just copy someone else's work – whatever you turn in should be what you think you understand.
- There is a web page for this course, which should be referred to for the most up-to-date information. The URL:
  http://www.hep.caltech.edu/~fcp/ph195/
- TA: Anura Abeyesinghe, anura@caltech.edu
- If you think a problem is completely trivial (and hence a waste of your time), you don't have to do it. Just write "trivial" where your solution would go, and you will get credit for it. Of course, this means you are volunteering to help the rest of the class understand it, if they don't find it so simple...

READING: Finish reading the "Density Matrix Formalism" course note.

PROBLEMS:

18. Let us try another example of the discussion we have been having in class concerning the use of the uncertainty relation on "localized" wave functions. Consider the three dimensional generalization. Hence, let $P(a)$ be the proability to find the particle, of mass m in a sphere of radius a centered at the origin.

    (a) Recall that in the one dimensional case, if the probability of finding the particle in the interval $(-a, a)$ was α, then a simple lower bound on the kinetic energy was obtained as:

$$T \geq \frac{1}{8m}\frac{\alpha^2}{a^2}. \tag{1}$$

Make a simple, but rigorous, generalization of this result to the three dimensional case. Don't worry about finding the "best" bound; even a "conservative" bound may be good enough to answer some questions of interest.

(b) We know that an atomic size is of order 10^{-10} m. Suppose that we have an electron which is known to be in a sphere of radius 10^{-10} m with 50% probability. What lower bound can you put on its kinetic energy? Is the result consistent with expectation; *e.g.*, with what you know about the kinetic energy of the electron in hydrogen?

(c) In ancient times, before the neutron was discovered, it was supposed that the nucleus contained both electrons and protons. A comfortable nuclear size is 5×10^{-15} m. Find a lower bound on the kinetic energy of an electron if the probability to be within this radius is 90%. If there is a problem with the validity of your bound, see if you can fix it.

(d) Now find a lower bound for a proton in the nucleus, if it has a probability of 90% to be within a region of radius 5×10^{-15} m.

19. Some more thoughts about time reversal: Exercise 13 of the "Ideas of Quantum Mechanics" course note.

20. The von Neumann mixing theorem: Exercise 3 of the "Density Matrix Formalism" course note.

21. Operators in product spaces: Exercise 4 of the "Density Matrix Formalism" course note.

22. Entropy in a two-state system: Exercise 5 of the "Density Matrix Formalism" course note.

23. Measuring the density matrix: Exercise 6 of the "Density Matrix Formalism" course note.