Physics 195a
Problem set number 5
Due 2 PM, Thursday, November 7, 2002

Notes about course:

- Homework should be turned in to the TA's mail slot on the first floor of East Bridge.
- Collaboration policy: OK to work together in small groups, and to help with each other's understanding. Best to first give problems a good try by yourself. Don't just copy someone else's work – whatever you turn in should be what you think you understand.
- There is a web page for this course, which should be referred to for the most up-to-date information. The URL: http://www.hep.caltech.edu/˜fcp/ph195/
- TA: Anura Abeyesinghe, anura@caltech.edu
- If you think a problem is completely trivial (and hence a waste of your time), you don't have to do it. Just write "trivial" where your solution would go, and you will get credit for it. Of course, this means you are volunteering to help the rest of the class understand it, if they don't find it so simple...

READING: Read the "The K^0: An Interesting Example of a 'Two-State' System" course note.

PROBLEMS:

24. Suppose we have a system with total angular momentum 1. Pick a basis corresponding to the three eigenvectors of the z-component of angular momentum, J_z, with eigenvalues $+1, 0, -1$, respectively. We are given an ensemble described by density matrix:

$$\rho = \frac{1}{4}\begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

(a) Is ρ a permissible density matrix? Give your reasoning. For the remainder of this problem, assume that it is permissible. Does it describe a pure or mixed state? Give your reasoning.

(b) Given the ensemble described by ρ, what is the average value of J_z?

(c) What is the spread (standard deviation) in measured values of J_z?

25. Coherent states with density matrices: Exercise 7 of the "Density Matrix Formalism" course note.

26. Density matrix for a spin 1/2 system in a magnetic field: Exercise 8 of the "Density Matrix Formalism" course note.

27. Entropy for a system of spin 1/2 particles in a magnetic field: Exercise 9 of the "Density Matrix Formalism" course note.

28. Hamiltonian in the particle-antiparticle basis: Exercise 1 of the K^0 course note.

29. Review of Schrödinger equation in three dimensions: Central potential problem. There are some areas of elementary quantum mechanics that I want to make sure don't fall through the cracks in your education, in particular, the central force problem and the specific case of the one-electron atom.

    Suppose we have two particles, of masses m_1 and m_2, described by position coordinates $\mathbf{x}_1$ and $\mathbf{x_2}$. Assume that they interact with each other via a potential $V(\mathbf{x}_1, \mathbf{x}_2)$.

    (a) Write down the Hamiltonian for this system. Show that it may be transformed to a description in terms of center-of-mass and relative coordinates. Show that the problem then reduces to two problems: one for the center-of-mass motion, and one for the relative motion, if the potential can be separated into a term depending only on the position of the center-of-mass, plus a term depending only on the relative locations of the particles. Now assume that the potential does not depend on the center-of-mass position, and solve for the center-of-mass motion. Is your solution sensible?

(b) Suppose V is a function of the separation between the two particles only, $V = V(|\mathbf{x}|)$, where $\mathbf{x} \equiv \mathbf{x}_1 - \mathbf{x}_2$. Solve the Schrödinger equation for the angular dependence and show that the Schrödinger equation may be reduced to an equivalent one dimensional problem. Give the "effective potential" for this equivalent one dimensional problem.