Physics 195b
Problem set number 11
Due 2 PM, Thursday, January 16, 2003

Notes about course:

- Homework should be turned in to the TA's mail slot on the first floor of East Bridge.
- Collaboration policy: OK to work together in small groups, and to help with each other's understanding. Best to first give problems a good try by yourself. Don't just copy someone else's work – whatever you turn in should be what you think you understand.
- There is a web page for this course, which should be referred to for the most up-to-date information. The URL: http://www.hep.caltech.edu/˜fcp/ph195/
- TA: Anura Abeyesinghe, anura@caltech.edu
- If you think a problem is completely trivial (and hence a waste of your time), you don't have to do it. Just write "trivial" where your solution would go, and you will get credit for it. Of course, this means you are volunteering to help the rest of the class understand it, if they don't find it so simple...

READING: Read sections 10 through 12 of the "Angular Momentum" course note.

PROBLEMS:

51. More on application of symmetry principles for selection rules: Charge Conjugation. Do Exercise 5 of the Angular Momentum course note.
52. Rotational and inversion symmetry, applied to determining selection rules: Exercise 10 of the Angular Momentum course note.
53. Reduction of a representation into irreducible representations: Exercise 11 of the Angular Momentum course note.
54. Little-d functions: Exercise 13 of the Angular Momentum course note.

55. We discussed Berry's phase in class, including the example of the Aharonov-Bohm effect. Now that we know more about rotations, let us try another example.

Consider a spin-1/2 particle of magnetic moment μ in a magnetic field, $\mathbf{B}$. Let the strength of the magnetic field be a constant, but suppose that its direction is slowly changing. Suppose the magnetic field vector is swept through a closed curve (*i.e.*, think of the tip of its vector as been varied along a closed curve on the surface of a sphere). Thus, two parameters describing the direction of the field are being varied, which we might take to be the polar angles (θ, ϕ).

(a) What does "slow" mean? That is, on what time scale should the variation be slow, in order for the adiabatic approximation to be reasonable?

(b) What is the expectation value of the spin vector in the adiabatic ground state?

(c) Express the adiabatic ground state wave function, $\psi^{(0)}_{\theta\phi}$, in terms of the adiabatically-varied parameters. You can achieve this by inspection from your result to part (b), but I urge you to try doing it instead (or, in addition, as a check) by performing a rotation on a vector to the desired polar angles, now that we know how to do this. I hope you'll even try it with our general rotation matrix formalism (the D matrices), even though you can short-cut this for spin-$\frac{1}{2}$.

(d) Suppose we let the $\mathbf{B}$ field direction rotate slowly around the 3-axis at constant θ. Calculate Berry's phase for this situation. [Recall, from first quarter, that Berry's phase is the change in the phase of the adiabatic wave function (ground state here) over one complete circuit in parameter space. We found that the Berry phase is given by:

$$\gamma_B = i \oint d\boldsymbol{\alpha} \cdot \langle \psi^{(0)}_{\boldsymbol{\alpha}} | \nabla_{\boldsymbol{\alpha}} \psi^{(0)}_{\boldsymbol{\alpha}} \rangle, \tag{116}$$

where $\boldsymbol{\alpha}$ is a vector in parameter space.] See if you can give a geometric interpretation to your answer, in terms of an amount of solid angle swept out by the circuit in parameter space.