Physics 195b
Problem set number 13
Due 2 PM, Thursday, January 30, 2003

Notes about course:

- Homework should be turned in to the TA's mail slot on the first floor of East Bridge.
- Collaboration policy: OK to work together in small groups, and to help with each other's understanding. Best to first give problems a good try by yourself. Don't just copy someone else's work – whatever you turn in should be what you think you understand.
- There is a web page for this course, which should be referred to for the most up-to-date information. The URL: http://www.hep.caltech.edu/˜fcp/ph195/
- TA: Anura Abeyesinghe, anura@caltech.edu
- If you think a problem is completely trivial (and hence a waste of your time), you don't have to do it. Just write "trivial" where your solution would go, and you will get credit for it. Of course, this means you are volunteering to help the rest of the class understand it, if they don't find it so simple...

READING: Read sections 1-5 of the "Approximate Methods" course note.

PROBLEMS:

61. Exercise 15 of the Angular Momentum course note.
62. Exercise 17 of the Angular Momentum course note.
63. More on decay angular distributions: Do exercise 19 of the Angular Momentum course note.
64. Let us discuss some issues relevant to the proof you gave of Yang's theorem (by the way, the original reference is: C. N. Yang, Physical Review **77** (1950) 242) in exercise 10 of the Angular Momentum course note.

(a) Let us consider the parity again, and try to establish the connection between how I expected you to do the homework, and an equivalent argument based on the identical boson symmetry.

In general, the parity of a system of two particles, when their state is an eigenstate of the parity operator, may be expressed in the form:

$$P = \eta_{\text{intrinsic}} \quad \eta_{\text{spatial}},$$

where $P^2 = 1$, $\eta_{\text{intrinsic}}$ refers to any intrinsic parity due to the interchange of the positions of the particles, and η_{spatial} refers to the effect of parity on the spatial part of the wave function. Given a system of two identical particles, and considering the action of parity on $Y_{L0}(\theta, \phi)$, determine the parity of the system for given orbital angular momentum L.

For a two-photon system, such as we considered in our discussion of Yang's theorem, explicitly consider boson symmetry (*i.e.*, that the wave function is invariant under interchange of all quantum numbers for two identical bosons) to once again determine the effect of parity of the states:

$$|\uparrow\uparrow\rangle, \quad |\downarrow\downarrow\rangle, \quad |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle, \quad |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Also, give the allowed possibilities for the orbital angular momentum for these states, at least at the level of our current discussion.

You may now wish to go back to your original derivation of Yang's theorem, and determine where you implicitly made use of the boson symmetry.

(b) Continuing our discussion of Yang's theorem, there may be some concern about the total spin angular momentum of the two photon states, and whether the appropriate values are possible to give the right overall angular momentum when combined with even or odd orbital angular momenta. Using a table of Clebsch-Gordan coefficients or otherwise, let us try to alleviate this concern. Thus, decompose our four 2-photon helicity states (with J_z values indicated by $|\uparrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$, $\frac{1}{\sqrt{2}}[|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle]$, $\frac{1}{\sqrt{2}}[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle]$, where the photons are travelling along the $+$ and $-z$ axis) into states of total spin angular momenta and spin projection along the z-axis: $| S, S_z\rangle$. Hence, show that a $J^P = 0^+$ particle may decay into two photons with relative orbital angular momentum $L = 2$ or 0, and a $J^P = 0^-$ particle may decay into two photons with relative angular momentum $L = 1$.

65. Do exercise 1 of the Approximate Methods course note.