Physics 195b
Problem set number 16
Due 2 PM, Thursday, February 20, 2003

Notes about course:

- Homework should be turned in to the TA's mail slot on the first floor of East Bridge.

- Collaboration policy: OK to work together in small groups, and to help with each other's understanding. Best to first give problems a good try by yourself. Don't just copy someone else's work – whatever you turn in should be what you think you understand.

- There is a web page for this course, which should be referred to for the most up-to-date information. The URL:
  http://www.hep.caltech.edu/~fcp/ph195/

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- If you think a problem is completely trivial (and hence a waste of your time), you don't have to do it. Just write "trivial" where your solution would go, and you will get credit for it. Of course, this means you are volunteering to help the rest of the class understand it, if they don't find it so simple. . .

READING: Finish reading the "Scattering" course note.

PROBLEMS:

75. Potential well/bump and spherical solutions: Do Exercise 2 of the Scattering course note.

76. The "fundamental" and "effective" cross sections: Do Exercise 4 of the Scattering course note.

77. Parity conservation and scattering amplitudes: Do Exercise 5 of the Scattering course note.

78. Resonant scattering of light on an atom: Do Exercise 6 of the Scattering course note.

79. When we talked about the hyperfine splitting in atoms, we mentioned that the magnetic dipole moment of the proton is:

$$\boldsymbol{\mu}_p = g_p \frac{e}{2m_p}\mathbf{s}_p, \tag{139}$$

with a measured magnitude corresponding to a value for the gyromagnetic ratio of $g_p = 2 \times (2.792847337 \pm 0.000000029)$. Recall also that I mentioned that the prediction of the Dirac equation for a point spin-1/2 particle is $g = 2$. We may understand the fact that the proton gyromagnetic ratio is not two as being due to its compositeness: In the simple quark model, the proton is made of three quarks, two "ups" (u), and a "down" (d). The quarks are supposed to be point spin-1/2 particles, hence, their gyromagnetic ratios should be $g_u = g_d = 2$ (up to higher order corrections, as in the case of the electron). Let us see whether we can make sense out of the proton magnetic moment.

The proton magnetic moment should be the sum of the magnetic moments of its constituents, and any moments due to their orbital motion in the proton. The proton is the ground state baryon, so we assume that the three quarks are bound together (by the strong interaction) in a state with no orbital angular momentum. By Fermi statistics, the two identical up quarks must have an overall odd wave function under interchange of all quantum numbers. We must apply this with a bit of care, since we are including "color" as one of these quantum numbers here.

Let us look a little at the property of "color". It is the strong interaction analog of electric charge in the electromagnetic interaction. However, instead of one fundamental dimension in charge, there are three color directions, labelled as "red" (r), "blue" (b), and "green" (g). Unitary transformations in this color space, up to overall phases, are described by elements of the group $SU(3)$, the group of unitary unimodular 3×3 matrices. Just like combining spins, we may combine three colors according to a Clebsch-Gordan series, with the result:

$$3 \times 3 \times 3 = 10 + 8 + 8 + 1. \tag{140}$$

We haven't studied this group, so this decomposition into irreducible representations of the product representation is probably new to you.

However, the essential aspect here is that there is a singlet in the decomposition. That is, it is possible to combine three colors in such a way as to get a color-singlet state, *i.e.*, a state with no net color charge. These are the states of physical interest for our observed baryons, according to a postulate of the quark model. After some thought (perhaps involving raising and lowering operators along different directions in this color space), you could probably convince yourself that the singlet state in the decomposition above must be antisymmetric under the interchange of any two colors. Assuming this is the case, write down the color portion of the proton wave function.

Now that you know the color wave function of the quarks in the proton, write down the spin wave function.

Since the proton is *uud* and its isospin partner the neutron is *ddu*, and $m_p \approx m_n$, let us make the simplfying assumption that $m_u = m_d$. Given the measured value of g_p, what does your model give for m_u? Recall that the up quark has electric charge 2/3, and the down quark has electric charge $-1/3$, in units of the positron charge.

Finally, use your results to predict the gyromagnetic moment of the neutron, and compare with observation.