Physics 195b
Problem set number 17 – Solution to Problem 81
Due 2 PM, Thursday, February 27, 2003

READING: Read the "Identical Particles" course note.

PROBLEMS:

80. High energy limit: Do Exercise 7 of the Scattering course note.

81. Consider the graph in Fig. 1.

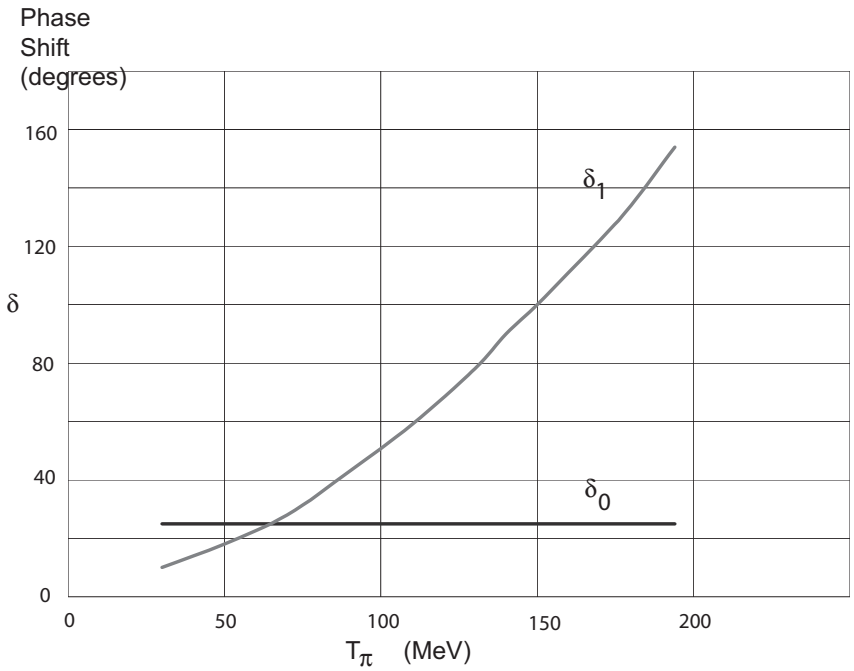

Figure 1: Made-up graph of phase shifts δ_0 and δ_1 for elastic $\pi^+ p$ scattering (neglecting spin).

Assume that the other phase shifts are negligible (*e.g.*, "low energy" is reasonably accurate). The pion mass and energy here are sufficiently small that we can at least entertain the approximation of an infinitely heavy proton at rest – we'll assume this to be the case, in any event. Note that T_π is the relativistic kinetic energy of the π^+ : $T_\pi = \sqrt{P_\pi^2 + m_\pi^2} - m_\pi$.

(a) Is the $\pi^+ p$ force principally attractive or repulsive (as shown in this figure)?

Solution: The phase shifts are positive, indicating a dominantly attractive potential.

(b) Plot the total cross section in mb (millibarns) as a function of energy, from T_π=40 to 200 MeV.

Solution: The total cross section in terms of the partial wave phase shifts is:

$$\sigma_T = \frac{4\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\sin^2\delta_\ell \tag{150}$$

$$= \frac{4\pi}{k^2}(\sin^2\delta_0 + 3\sin^2\delta_1). \tag{151}$$

The kinetic energy T_π is related to k by $T_\pi = \sqrt{m_\pi^2 + k^2} - m_\pi$, or

$$k = \sqrt{T(T+2m_\pi)}. \tag{152}$$

To convert to millibarns, we multiply by:

$$1 = (197\,\text{MeV-fm})^2 10\,\text{mb/fm}^2 = 3.88\times10^5\,\text{MeV}^2\text{mb}. \tag{153}$$

(c) Plot the angular distribution of the scattered π^+ at energies of 120, 140 and 160 MeV.

Solution:

$$\frac{d\sigma}{d\Omega} = |\frac{1}{2ik}\sum_{j=0}^{\infty}(2j+1)\left[e^{2i\delta_j(k)}-1\right]P_j(\cos\theta)|^2 \tag{154}$$

$$= \frac{1}{4k^2}|e^{2i\delta_0(k)} - 1 + 3(e^{2i\delta_j(k)}-1)\cos\theta|^2 \tag{155}$$

$$= \frac{1}{4k^2}\left\{[\cos\delta_0 - 1 + 3(\cos\delta_1 - 1)\cos\theta]^2 + [sin\delta_0 + 3\sin\delta_1\cos\theta]^2\right\}.$$

(d) What is the mean free path of 140 MeV pions in a liquid hydrogen target, with these "protons"?

Solution: The cross section for 140 MeV pions is ~ 260 mb. The density of liquid hydrogen is 0.0708 g/cm^3. The number density is $\rho == 4.2\times10^28\,\text{m}^{-3}$. The mean free path is thus

$$\lambda = \frac{1}{\sigma_T\rho} = 0.9\,\text{m}. \tag{156}$$

82. Inelastic scattering: Do Exercise 8 of the Scattering course note.

83. Exclusion principle and atomic states: Do Exercise 1 of the Identical Particles course note.