Physics 195b
Problem set number 19 – Solution to Problem 91
Due 2 PM, Thursday, March 13, 2003

READING: Read the "Second Quantization" course note.

PROBLEMS:

88. Probability current of a charged particle in an electromagnetic field: Do Exercise 2 of the Electromagnetic Interactions course note.

89. The electric field in quantum mechanics: Do Exercise 3 of the Electromagnetic Interactions course note.

90. Two-level fermion system, second quantized: Do Exercise 1 of the Second Quantization course note.

91. We have been discussing superconductivity as an application of quantum mechanics. We considered a very simple model at first, in order to demonstrate the plausibility of the formation of Cooper pairs. The scanned notes are available as a link from the Ph 195 home page. The first seven pages are of present interest.

    (a) Try to make an estimate for how large the Cooper pair is, perhaps by evaluating $\langle r\rangle$ or $\sqrt{\langle r^2\rangle}$.

    Solution: Let's try a simple uncertainty principle argument for a starter: We know the temperature scale, T_c at which materials normally become superconducting. The typical interaction energy in a superconductor may be expected to be of this same scale. Thus, the uncertainty in the momentum is of order

$$\Delta k \approx \frac{T_c}{\varepsilon_F} k_F, \tag{169}$$

where k_F is the momentum at the Fermi surface, and $\epsilon_F = k_F^2/2m$. Thus, we anticipate a typical spatial extent of order

$$\Delta x \approx \frac{1}{\Delta k} \approx \frac{\varepsilon}{T_c}\frac{1}{k_F}. \tag{170}$$

(b) Turn your estimate into a number, *e.g.*, comparing the size with the lattice spacing of the superconductor. You will no doubt need to make plausible estimates (guesses?) of unknown parameters.

Solution: We need to estimate the energy of the Fermi surface. We obtained in class that the momentum is given by:

$$k_F = (3\pi^2 n)^{1/3}, \tag{171}$$

where n is the number density of conduction electrons. What is this density, numerically? Well, in a conductor, we have of order one electron contributed to the conduction band by each atom. Consider niobium, with atomic weight 93 and density 8.6 g/cc. We estimate the number density as:

$$n \approx \frac{8.6}{93} 6 \times 10^{23} \approx 6 \times 10^{22}\,\text{electrons/cc} \tag{172}$$

Thus,

$$k_F \approx (180 \times 10^2 2)^{1/3} \approx 1.2 \times 10^8\,\text{cm}^{-1} \tag{173}$$

In other units (using $1 = 2 \times 10^{-5}$ eV-cm, this is $k_F \approx 2 \times 10^3$ eV. This corresponds to an energy of

$$\varepsilon_F \approx \frac{(2 \times 10^3)^2}{2 \times 0.5 \times 10^6} \approx 4\,\text{eV}. \tag{174}$$

Let us take a value of $T_c = 10$ K as a superconducting transition temperature for our estimate. This is approximately $\frac{10}{300}\frac{1}{40} \approx 10^{-3}$ eV. Thus,

$$\Delta x \approx \frac{1}{10^8}\frac{4}{10^{-3}} \approx 4 \times 10^{-5}\,\text{cm}. \tag{175}$$

If the lattice spacing is of order 10^{-8} cm, then this distance corresponds to roughly 4000 lattice spacings.