Physics 231a
Problem Set Number 4
Due Wednesday, October 27, 2004

Note: Some problems may be "review" for some of you. I am deliberately including problems which are potentially in this category. If the material of the problem is already well-known to you, such that doing the problem would not be instructive, just write "been there, done that", or suitable equivalent, for that problem, and I'll give you credit.

20. Standard Model review(?):
    (a) Compute, in lowest non-zero order perturbation theory, the QED differential cross section $d\sigma/d\Omega$ for $e^+e^- \to \mu^+\mu^-$, in the center-of-mass frame. [If you are already proficient in Feynman graph computations, but have not done this particular calculation, please do it – we'll want the result for other things].
    (b) From your answer to part (a), compute the total $e^+e^- \to \mu^+\mu^-$ scattering cross section in QED.
    (c) Give the differential and total cross sections for this process at high energy.

21. Considering isospin and the extended Pauli principle in which flavor is treated as another quantum number a quark (or a nucleon) may have, make an argument for the spin of the deuteron. Now make an equivalent argument which doesn't make use of the extended Pauli principle, hopefully leading to the same conclusion... Compare with experiment.

22. Consider the isotopic spin singlet $\eta$ meson:
    (a) The $\eta$ decays to $\gamma\gamma$ 39% of the time and to $\pi^+\pi^-\pi^0$ 23% of the time. Considering these two facts, is $G$-parity conserved?
    (b) The $\eta$ decays to $\pi^0\pi^0\pi^0$ 32% of the time. Considering this fact, is isospin conserved?

23. Certain neutral particles (*e.g.*, $\pi^0, \eta, \eta'$) are observed to decay into two photons, and others (*e.g.*, $\omega, \phi, \psi$) are not. Let us investigate the selection rules implied by angular momentum and parity conservation

(satisfied by electromagnetic and strong interactions) for the decay of a particle (call it "X") into two photons. Thus, we ask the question, what angular momentum $J$ and parity $P$ states are allowed for two photons?

Set up the problem in the center-of-mass frame of the two photons, with the z-axis in the direction of one photon. We know that since a photon is a massless spin-one particle, it has two possible spin states, which we can describe by its "helicity", *i.e.*, its spin projection along its direction of motion, which can take on the values $\pm 1$. Thus, a system of two photons can have the spin states:

$$|\uparrow\uparrow\rangle,\ |\downarrow\downarrow\rangle,\ |\uparrow\downarrow + \downarrow\uparrow\rangle,\ |\uparrow\downarrow - \downarrow\uparrow\rangle$$

(The first arrow refers to the photon in the $+z$ direction, the second to the photon in the $-z$ direction, and the direction of the arrow indicates the spin component along the $z$-axis, NOT to the helicity.) We consider the effect on these states of three operations (which, by parity and angular momentum conservation, should commute with the Hamiltonian):

- $P$: parity – reverses direction of motion of a particle, but leaves its angular momentum unaltered.
- $R_z(\alpha)$: rotation by angle $\alpha$ about the $z$-axis. A state with a given value of $J_z$ ($z$-component of angular momentum) is an eigenstate, with eigenvalue $e^{i\alpha J_z}$.
- $R_x(\pi)$: rotation by $\pi$ about $x$-axis. For our two photons, this reverses the direction of motion and also the angular momentum of each photon. For our "X" particle, this operation has the effect corresponding to the effect on the spherical harmonic with the appropriate eigenvalues:

  $$R_x(\pi) Y_{JJ_z}(\Omega)$$

  (Note that the $Y_{lm}$ functions are sufficient, since a fermion obviously can't decay into two photons and conserve angular momentum – hence X is a boson, and we needn't consider $\frac{1}{2}$-integer spins.)

Make sure that the above statements are intuitively clear to you.

(a) By considering the actions of these operations on our two-photon states, complete the following table: (one entry is filled in for you)

| Photonic Spin State | Transformation $P$ | $R_z(\alpha)$ | $R_x(\pi)$ |
|---|---|---|---|
| $\lvert\uparrow\uparrow\rangle$ | $+\lvert\uparrow\uparrow\rangle$ | | |
| $\lvert\downarrow\downarrow\rangle$ | | | |
| $\lvert\uparrow\downarrow+\downarrow\uparrow\rangle$ | | | |
| $\lvert\uparrow\downarrow-\downarrow\uparrow\rangle$ | | | |

(b) Now fill in a table of eigenvalues for a state (*i.e.*, our particle "X") of arbitrary integer spin $J$ and parity $P$ (or, if states are not eigenvectors, what the transformations yield):

| Spin $J$ | Transformation $P$ | $R_z(\alpha)$ | $R_x(\pi)$ |
|---|---|---|---|
| 0 | $\left\{\begin{matrix}+1\\-1\end{matrix}\right\}$ | | |
| 1 | $\left\{\begin{matrix}+1\\-1\end{matrix}\right\}$ | | |
| $2,4,6,\ldots$ | $\left\{\begin{matrix}+1\\-1\end{matrix}\right\}$ | | |
| $3,5,7,\ldots$ | $\left\{\begin{matrix}+1\\-1\end{matrix}\right\}$ | | |

Note that there may be more than one eigenvalue of $R_z(\alpha)$ for a given row, corresponding to the different possible values of $J_z$.

(c) Finally, by using your answers to parts (a) and (b), determine the allowed and forbidden $J^P$ states decaying into two photons, and the appropriate photonic helicity states for the allowed transitions. Put your answer in the form of a table:

| Parity | Spin 0 | 1 | 2,4,... | 3,5,... |
|---|---|---|---|---|
| +1 | | | | |
| −1 | $\lvert\uparrow\downarrow-\downarrow\uparrow\rangle$ | | | |

You have (I hope) just derived something which is often referred to as "Yang's theorem". Note: People often get this wrong, so be careful!

24. We have discussed the pseudoscalar and vector meson "nonets".

(a) Draw an $I_3 - Y$ diagram for the lowest radial state tensor nonet, complete with observed particle assignments to the various sites.

(b) Give the physical states for the $I_3 = Y = 0$ particles in terms of quark content, including the observed mixing.