

Physics 231a
Problem Set Number 6
Due Wednesday, November 10, 2004

Note: Some problems may be “review” for some of you. I am deliberately including problems which are potentially in this category. If the material of the problem is already well-known to you, such that doing the problem would not be instructive, just write “been there, done that”, or suitable equivalent, for that problem, and I’ll give you credit.

30. Standard Model Review(?): Last week we illustrated the essential features of the Higgs mechanism using a $U(1)$ gauge theory. Now let us carry out the program for the physically more relevant case of $SU(2)$. Thus, I am asking you to repeat problem 25 for $SU(2)$.

Do not worry about interactions with fermions yet; just concentrate on the boson fields. Call the $SU(2)$ gauge fields $W_i, i = 1, 2, 3$, where this set of fields transforms as a triplet (e.g., weak isospin equal to one) under $SU(2)$.

Note that you will have to deal with the complication, even without the Higgs mechanism, of the fact that $SU(2)$ is non-abelian. You may refer to your answer to the QCD problem (14) to help with this. Thus, to start, you should write down the Lagrangian for a massless $SU(2)$ gauge theory (omitting any fermion terms).

The goal of this problem is to write down the Lagrangian with Higgs and W fields, such that the W fields have acquired a mass via the Higgs mechanism. Since you are trying to give mass to three fields, you will need more scalar degrees of freedom than you considered last week. You are asked to use the choice in the standard model, of an $SU(2)$ doublet of complex scalar fields:

$$\phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}. \quad (35)$$

When you are done, draw all of the Feynman graph vertices that exist in your Lagrangian.

31. In class and problem 28, we discussed the neutral kaon (K) meson, in particular the phenomenon of $K^0 - \bar{K}^0$ mixing. Let us think about

this system a bit further. The K^0 and \bar{K}^0 mesons interact in matter, dominantly via the strong interaction. Approximately, the cross section for an interaction with a deuteron is:

$$\sigma(K^0 d) = 36 \text{ millibarns} \quad (36)$$

$$\sigma(\bar{K}^0 d) = 59 \text{ millibarns}, \quad (37)$$

at a kaon momentum of, say, 1.5 GeV. Note that a “barn” is a unit of area equal to 10^{-24} cm^2 .

- (a) Consider a beam of kaons (momentum 1.5 GeV) incident on a target of liquid deuterium. Let λ be the K^0 “interaction length”, *i.e.*, the average distance that a K^0 will travel in the deuterium before it interacts according to the above cross section. Similarly, let $\bar{\lambda}$ be the \bar{K}^0 interaction length. To a good enough approximation for our purposes, you may treat the deuterium as a collection of deuterons (why?). The density of liquid deuterium is approximately $\rho = 0.17 \text{ g/cm}^3$. What are λ and $\bar{\lambda}$, in centimeters?
- (b) Suppose we have prepared a beam of K_L^0 mesons, *e.g.*, by first creating a K^0 beam and waiting long enough for the K_S^0 component to decay away. If we let this K_L^0 beam traverse a deuterium target, the K^0 and \bar{K}^0 components will interact differently, and we may end up with some K_S^0 mesons exiting the target. Let us make an estimate for the size of this effect.

Since the kaon is relativistic, we need to be a little careful: In the K_L^0 rest frame, the amplitude depends on time t^* according to:

$$\exp(-im_L t^* - \Gamma_L t^*/2), \quad (38)$$

where $\Gamma_L = 1/\tau_L$ is the K_L^0 decay rate. In the laboratory frame, where the kaon is moving with speed v , and $\gamma = 1/\sqrt{1-v^2}$, $t^* \rightarrow t/\gamma$, where t is the time as measured in the laboratory frame. In the lab frame, we have $t/\gamma = x/\gamma v$, and we may write the amplitude as for the K_L^0 as:

$$\exp(-im_L x/\gamma v - \Gamma_L x/2\gamma v), \quad (39)$$

Let us consider a deuterium target, of thickness w , along the beam direction. At a distance x into the target, an interaction may

occur, resulting in a final state:

$$\frac{1}{\sqrt{2}}(f|K^0\rangle - \bar{f}|\bar{K}^0\rangle), \quad (40)$$

where, for example, the amplitude f for the K^0 component traversing distance dx is just:

$$f = e^{-dx/2\lambda} \approx 1 - \frac{dx}{2\lambda}. \quad (41)$$

Put all this together and find an expression for the probability to observe a K_S^0 to emerge from the deuterium, for a K_L^0 incident. Assume that $w \ll \lambda$. You may wish to use $\Delta m \equiv m_L - m_S$, $\Gamma_{S,L} \equiv 1/\tau_{S,L}$, and $\Delta\Gamma \equiv \Gamma_L - \Gamma_S \approx -\Gamma_S$

- (c) Suppose $w = 10$ cm and $\gamma v = 3$. What is the probability to observe a K_S^0 emerging from the target? What is the probability to observe a K_L^0 ? You may use:

$$\Gamma_S = 1.1 \times 10^{10} \text{ s}^{-1}, \quad (42)$$

$$\Delta m = 0.5 \times 10^{10} \text{ s}^{-1}. \quad (43)$$

You have been investigating a phenomenon often called “regeneration” – by passing through material, a K_S^0 component to the beam has been “regenerated”. A similar consideration has been proposed to help explain the “solar neutrino problem”.

32. We mentioned the “bag model” briefly in class. Let’s get a little intuition on the basic idea.

In non-relativistic potential quark model calculations of hadron spectroscopy, which are somewhat successful, depending on circumstances, the sum of the constituent (valence) quark masses is of the order of the mass of the hadron. For example, a proton, with a mass of 1 GeV, is made of two ups and a down. A neutron, with the same mass, is made of two downs and an up, with a similar wave-function. Thus, in such a model, the up and down quarks have masses of order 300 MeV.

We will soon find, however, that when one probes deep inside the hadron (“deep inelastic scattering”), the quarks appear to have much smaller masses. Such scattering processes are largely describable in

terms of perturbation theory, and these masses are hence the masses which appear in the Lagrangian (“current masses”). Can we reconcile these two different pictures of the quark mass?

Consider a phenomenological model of a hadron, in which the N constituent quarks are massless, but are confined to some region of radius a . There is an energy density associated with this confinement. Suppose the average energy density in the confinement region (“bag”) is B . The energy (*i.e.*, mass in its rest frame) of the hadron is given by the kinetic energies of the N quarks, plus the bag energy.

- (a) Determine the dependence of energy, E , on a , N , and B , and, by requiring stability, determine the relation between the hadron mass and the radius of the bag. This, of course, is a crude model, so don’t worry too much about all the numerical factors – try to get an answer for the radius that you might expect to work at the factor of two level.
- (b) Calculate the radius for the proton, in fm. Let us see whether your result is plausible: The (strong) cross section for pp scattering at tens of GeV is around 40 mb. What proton radius does this imply? Compare with your bag model calculation. Finally, calculate the value of B , in MeV/fm³.

The interpretation of all this is that we can view the constituent quark mass as arising from the kinetic and confining potential energy: The constituent mass is an “effective mass”, not unlike the effective mass of an electron moving in the field of a crystal. I hope that you have made plausible that a massless quark confined to a region of order 1 fm acquires an effective mass of order 300 MeV. We should ultimately be careful how we use this bag model: The dynamics may affect the constituent masses, and hence, the constituent masses may be different, *e.g.*, for baryons and mesons.

33. In problem 20 you calculated the cross section for $e^+e^- \rightarrow \mu^+\mu^-$ via 1 photon exchange (lowest-order perturbation theory in QED).

You have all you need to calculate the cross section for $e^+e^- \rightarrow$ hadrons, at least approximately. Let us see how this important result is obtained very simply from what you already know. First, note that e^+e^- annihilation is a QED process – it will be the electric charges of the hadrons

that matter. Second, note that hadrons are “really” made of quarks. Thus, at least at high enough energies, what is really happening is that we have $e^+e^- \rightarrow q\bar{q}$, and that the quarks, on trying to “separate”, involve the strong interaction to evolve into some hadronic final state. This suggests that all we really need to do is calculate the QED process $e^+e^- \rightarrow q\bar{q}$, since we know the quarks will turn into physical hadrons, and we don’t care which hadronic final state we get. The lowest order diagram we need, then, is quite familiar.

It is often convenient (and very common), to give not the hadronic cross section, but the ratio, R , of the hadronic cross section to the $e^+e^- \rightarrow \mu^+\mu^-$ cross section via single photon exchange:

$$R \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{(\frac{4\pi}{3}\alpha^2/s)}$$

Using the work we have already done to get the lowest-order contribution to $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$, calculate what R should be, also to lowest order in QED. Assume that you are above the threshold to produce $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, $c\bar{c}$ and $b\bar{b}$. The experimental data is summarized by the Particle Data Group in Figures 40.6–40.8 of the Review of Particle Properties, see: <http://pdg.lbl.gov/2004/reviews/hadronicrpp.pdf> Compare your result with the data points in the figures for $E_{cm} \sim 20 - 40$ GeV, say. Do you agree? If not, would a hint about color help?

34. In problem 15 you investigated the possibility that the $\pi^+ - \pi^0$ mass difference might be of Coulombic origin. Another possible contribution is from the spin-spin magnetic moment interaction between the quark and anti-quark. Recall that the hyperfine energy shift is given by:

$$\Delta E_{\text{hf}} = -\frac{2}{3}\boldsymbol{\mu}_1 \cdot \boldsymbol{\mu}_2 |\psi(0)|^2.$$

From the Dirac equation, the magnetic moment of a spin-1/2 Dirac fermion is:

$$\boldsymbol{\mu} = Q \frac{e}{2m} \boldsymbol{\sigma}.$$

- (a) Obtain a formula for the $\pi^+ - \pi^0$ mass difference from this source. Calculate the value of the square of the wave function at the origin, for the observed mass splitting. Use $m_u \approx m_d \approx 340$ MeV, as

determined by fitting the proton magnetic moment in the valence quark model (*e.g.*, see Halzen and Martin). Express your answer as $(x \text{ fm}^{-1})^3$ and as $(y \text{ MeV})^3$.

- (b) Critically evaluate whether what you have just done is plausible. For example, you might think about the kaons, and about the ρ .