Physics 231b
Problem Set Number 11
Due Wednesday, January 12 2005

Note: Some problems may be "review" for some of you. I am deliberately including problems which are potentially in this category. If the material of the problem is already well-known to you, such that doing the problem would not be instructive, just write "been there, done that", or suitable equivalent, for that problem, and I'll give you credit.

55. Standard Model Review(?): We have seen that by introducing an $SU(2)$ doublet of complex scalar fields we can give masses to the weak gauge bosons via the Higgs mechanism. Unfortunately, we pay the price of predicting the existence of a physical Higgs scalar particle, for which there is so far no experimental evidence. We might hope to concoct a scheme to give the gauge bosons mass without paying this price. Since we have three gauge bosons, we need three degrees of freedom to give them all masses and therefore longitudinal degrees of freedom. Hence consider a model with an $SU(2)$ triplet of real scalar fields, $\phi^T = (\phi_1, \phi_2, \phi_3)$, with potential:

$$V(\phi) = \mu^2 \phi^T \phi + \lambda (\phi^T \phi)^2,$$

where $\mu^2 < 0$, and $\lambda > 0$. Does this work?

56. In class, we asserted that the most general form for the proton electromagnetic current in ep elastic scattering is:

$$J^\mu(x) = e\bar{u}(p') \left[F_1(q^2)\gamma^\mu + \frac{\kappa}{2M} F_2(q^2) i\sigma^{\mu\nu} q_\nu \right] u(p) e^{i(p'-p)\cdot x},$$

where p is the proton's initial 4-momentum, p' its final 4-momentum, and $q = p' - p$. Justify this assertion, *i.e.*, explain why the other possible Lorentz 4-vectors constructable from p, p', and the gamma matrices don't appear. This is a typical sort of procedure – write down the most general thing you can, then figure out what terms are really independent, and what contributions really may be present given constraints such as current conservation. The "form factors" F_1 and F_2 parameterize our remaining ignorance of the proton structure.

57. The Breit Frame: Let us try to get some further intuition concerning the elastic from factors G_E and G_M.

(a) Show that the proton current of the previous problem can be rewritten in the form:

$$J^\mu(x) = e\bar{u}(p')\left[(F_1 + \kappa F_2)\gamma^\mu - \frac{(p^\mu + p'^\mu)}{2M}\kappa F_2\right]u(p)e^{i(p'-p)\cdot x},$$

(b) The Breit frame, or "brick wall" frame is defined as the frame in which $\mathbf{p}' = -\mathbf{p}$. Choose $\mathbf{p}$ to be along the z axis. Compute the proton current in this frame for the different possible helicity states, and hence relate the charge density ρ, and current density $\mathbf{J}$ to G_E and G_M.

58. In class, we wrote down the high energy differential cross section, in the laboratory frame, for inelastic $ep \to eX$ scattering:

$$\frac{d\sigma}{dE'd\Omega} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\left[W_2(\nu, Q^2)\cos^2\frac{\theta}{2} + 2W_1(\nu, Q^2)\sin^2\frac{\theta}{2}\right].$$

Show that this may be rewritten in the frame-invariant form:

$$M\nu_{\max}\frac{d\sigma}{dxdy} = \frac{2\pi\alpha^2}{x^2y^2}\left\{xy^2F_1 + \left[(1-y) - \frac{Mxy}{2\nu_{\max}}\right]F_2\right\}.$$

59. When I compared the interaction time with the time scale for parton interaction within the proton wavefunction in class, I made a simple estimate for the latter time scale. In particular, I argued that this time scale should be of the order of the size of the proton. We may also estimate this time scale from a somewhat different perspective: We consider the proton wavefunction to consist of a superposition of virtual states, described by a parton (the "struck" parton, in particular) carrying momentum fraction x, and the remainder of the partons carrying fraction $1-x$. As we noted in class, except in the ∞-momentum frame, such a configuration violates energy conservation, because of the finite mass of the partons, and because of their finite transverse momenta within the proton. Thus, such a state must have a lifetime consistent with the "uncertainty relation" $\Delta E\Delta t \gtrsim 1$.

Let us suppose that we have a proton of momentum $\mid \vec{p} \mid = p \gg M$, where M is the proton mass. Imagine a configuration where a parton with rest mass m_1 and transverse momentum $q_{\perp 1}$ carries a fraction x of the parton's momentum. For convenience, lump whatever else there is into a "parton" of mass m_2, transverse momentum $q_{\perp 2}$ and longitudinal momentum fraction $(1-x)$. Show that an estimate for the lifetime of a virtual state is given by:

$$T \cong \frac{2P}{m_{\perp 1}^2/x + m_{\perp 2}^2/(1-x) - M^2}$$

where the "transverse masses" $m_\perp$ are given by:

$$m_{\perp 1}^2 = m_1^2 + q_{\perp 1}^2$$

$$m_{\perp 2}^2 = m_2^2 + q_{\perp 2}^2$$

Note that this estimate is not in disagreement (see if you can convince yourself) with the estimate given in class, except possibly for the very short-lived limiting cases $x \to 0$ and $x \to 1$.