

Physics 231b
Problem Set Number 12
Due Wednesday, January 19 2005

Note: Some problems may be “review” for some of you. I am deliberately including problems which are potentially in this category. If the material of the problem is already well-known to you, such that doing the problem would not be instructive, just write “been there, done that”, or suitable equivalent, for that problem, and I’ll give you credit.

59. (Carried over from last week) When I compared the interaction time with the time scale for parton interaction within the proton wavefunction in class, I made a simple estimate for the latter time scale. In particular, I argued that this time scale should be of the order of the size of the proton. We may also estimate this time scale from a somewhat different perspective: We consider the proton wavefunction to consist of a superposition of virtual states, described by a parton (the “struck” parton, in particular) carrying momentum fraction x , and the remainder of the partons carrying fraction $1 - x$. As we noted in class, except in the ∞ -momentum frame, such a configuration violates energy conservation, because of the finite mass of the partons, and because of their finite transverse momenta within the proton. Thus, such a state must have a lifetime consistent with the “uncertainty relation” $\Delta E \Delta t \gtrsim 1$.

Let us suppose that we have a proton of momentum $|\vec{p}| = p \gg M$, where M is the proton mass. Imagine a configuration where a parton with rest mass m_1 and transverse momentum $q_{\perp 1}$ carries a fraction x of the parton’s momentum. For convenience, lump whatever else there is into a “parton” of mass m_2 , transverse momentum $q_{\perp 2}$ and longitudinal momentum fraction $(1 - x)$. Show that an estimate for the lifetime of a virtual state is given by:

$$T \cong \frac{2P}{m_{\perp 1}^2/x + m_{\perp 2}^2/(1 - x) - M^2}$$

where the “transverse masses” m_{\perp} are given by:

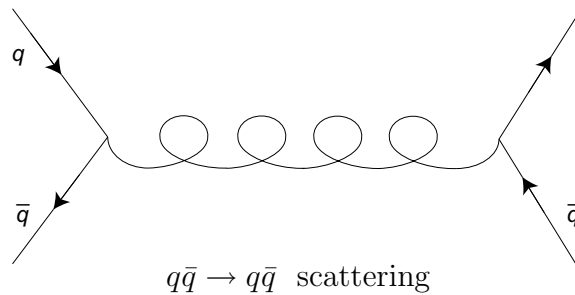
$$m_{\perp 1}^2 = m_1^2 + q_{\perp 1}^2$$

$$m_{\perp 2}^2 = m_2^2 + q_{\perp 2}^2$$

Note that this estimate is not in disagreement (see if you can convince yourself) with the estimate given in class, except possibly for the very short-lived limiting cases $x \rightarrow 0$ and $x \rightarrow 1$.

60. Standard Model Review(?):

We consider the calculation of the following graph in QCD perturbation theory:



- (a) Write down the Lorentz-invariant matrix element for this graph, corresponding to a quark with color index i plus an antiquark with index j scattering into a quark with index m and antiquark with index n .
- (b) Working in the Gell-Mann representation, assume that index 1 is “red”, 2 is “green”, and 3 is “blue”. Calculate the cross section, according to the above graph, for red plus anti-red to scatter into red plus anti-red; $q_r\bar{q}_r \rightarrow q_r\bar{q}_r$ or simply, $r\bar{r} \rightarrow r\bar{r}$. Neglect the quark masses. Note that the initial and final states must be **flavor**-singlets (not **color**), but otherwise the initial and final quarks may have different flavors. Put your answer in terms of the “strong coupling constant” $\alpha_s = \frac{g^2}{4\pi}$. You should realize that there is almost no work involved in doing this problem, since we did a very similar problem in QED. Except for the color indices, the calculation is identical, so all you really need to do is figure out how color modifies the QED calculation.

I hope it is clear that this is a general feature – at least for graphs containing only $q\bar{q}G$ vertices, if a similar graph for QED has already been calculated, then simple modifications may be applied to obtain the result for the QCD graph.

- (c) Calculate the cross section, according to the above graph, for $r\bar{r} \rightarrow \frac{1}{\sqrt{3}}(r\bar{r} + g\bar{g} + b\bar{b})$. Make sure you understand your answer.
61. Consider photon-hadron scattering at high energies, in the parton model. Let σ_T be the cross section for the scattering of transversely polarized photons, and let σ_L be the cross section for the scattering of longitudinally polarized photons. [The photon, possibly produced with a lepton, is absorbed by the parton in this model].
- (a) Suppose the parton is spinless. Show, by a simple argument, that the cross section $\sigma_T \rightarrow 0$ in the high energy limit.
- (b) Now suppose the parton is spin-1/2. Again, produce a simple argument for the value of σ_L/σ_T in the high energy limit. The result, which bears up in more detailed calculation, is implied by the Callan-Gross relation.
62. Assuming the quark model, derive x -independent bounds on the ratio:

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)}.$$

What physical circumstance do the upper and lower bounds correspond to?

It is perhaps worth noting that the quark model, with fractionally charged quarks, may not be the only model which can give these results. We may digress on this point.

63. Inclusive particle production: The notion of a “bremsstrahlung spectrum” pops up in other ways besides those alluded to in class. Let us consider briefly the evidently very difficult question of particle production in hadron-hadron collisions and see that we can extract some very useful general features. In particular, we concern ourselves with the following inclusive cross section: What is the behavior of the single particle inclusive cross section $\frac{E d^3\sigma}{d^3(\vec{p})}$ [recall that $\frac{d^3(\vec{p})}{E}$ is Lorentz-invariant, hence this is a Lorentz-invariant cross section for colinear collisions] in the process $A + B \rightarrow C + \text{anything}$? E, \vec{p} refer to the energy and momentum of particle C . It is useful to consider the transverse (p_T) and longitudinal (p_{\parallel}) momenta separately [and we consider only “high” center-of-mass energy collisions].

We may argue that the transverse momentum distribution of C is governed principally by the size of the hadrons involved, (at least, for p_T not so large that we are observing hard parton-parton scattering) hence, the p_T distribution should be approximately independent of $s = E_{cm}^2$ and of p_{\parallel} . Since the typical hadronic size is of order $\lesssim 1$ fm, we expect $\langle p_T \rangle \sim$ few hundred MeV, as is observed experimentally. For p_T not too large, $d\sigma/dp_T^2$ falls approximately exponentially:

$$\frac{d\sigma}{dp_T^2} \sim \exp[-p_T/350 \text{ MeV}].$$

The behavior of the longitudinal momentum distribution is perhaps more subtle. Let us define the longitudinal “scaling” variable

$$x = p_{\parallel}/(p_{\parallel})_{max} \simeq \frac{2}{\sqrt{s}}p_{\parallel}$$

This is known as “Feynman- x ”, and is similar in appearance, but not the same thing as, the x -variable we have been using in our parton-model discussions in class. Let us suppose that, indeed, we have (at least for x not too large) a bremsstrahlung spectrum in x :

$$d^3\sigma \approx f(\vec{p}_T) d^2p_T \frac{dx}{x}$$

- (a) Show that (integrating over d^2p_T), this yields the result:

$$\frac{d\sigma}{dy} = \text{constant}$$

where,

$$y = \frac{1}{2} \ln \left(\frac{E + p_{\parallel}}{E - p_{\parallel}} \right) = \ln \left(\frac{E + p_{\parallel}}{m_T} \right)$$

is the rapidity of the particle C ($m_T =$ “transverse mass” = $\sqrt{m_C^2 + p_T^2}$) [recall Ph231a, problem 2].

- (b) For large enough s this bremsstrahlung contribution dominates, and hence show that the number of particles in the final state (“multiplicity”) grows with s as

$$\langle n \rangle = A \ln s \quad (+ \text{ a constant})$$

Note that what we have done here is describe the one-particle inclusive cross section for the “typical” (*i.e.*, excluding the very hard collisions between constituent partons) inelastic hadron-hadron scattering at high energy:

$$\frac{d^3\sigma}{dp_T^2 dy} \sim e^{-ap_T}$$

We have neglected some issues, such as the possibility of other sources of s dependence. However, this picture is a reasonable description of such events (often called “minimum bias events”, which I have mentioned before) and is very useful in making estimates. We have been dealing with “Feynman scaling”: R. P. Feynman, *Phys. Rev. Lett.* **23**, 1415 (1969).