Physics 231b
Problem Set Number 13
Due Wednesday, January 26, 2005

Note: Some problems may be "review" for some of you. I am deliberately including problems which are potentially in this category. If the material of the problem is already well-known to you, such that doing the problem would not be instructive, just write "been there, done that", or suitable equivalent, for that problem, and I'll give you credit.

64. Standard Model Review(?): Determine the vertex factors to be used in perturbation theory for the Higgs boson couplings to the weak gauge bosons. Remember that trilinear and quadrilinear vertices exist.

65. In problem 63, you investigated Feynman scaling, and showed that the average multiplicity of particles in the final state should increase approximately as $\ln s$. Let us pursue this idea further, and ask what the multiplicity distribution should be. We thus consider a model, motivated by the bremsstrahlung hypothesis, in which particles are emitted independently. Suppose that the total rapidity (kinematic limits) range is Y. Suppose further that, in a frame with the observer at rapidity 0, the probability of emission of a particle with smallest (positive) rapidity y is given by $\exp(-y/\Delta y)$.

    (a) In what sense does this supposition satisfy our hypothesis of independent emission? What is the probability $P(n)$ that n particles will be produced? What is Δy?

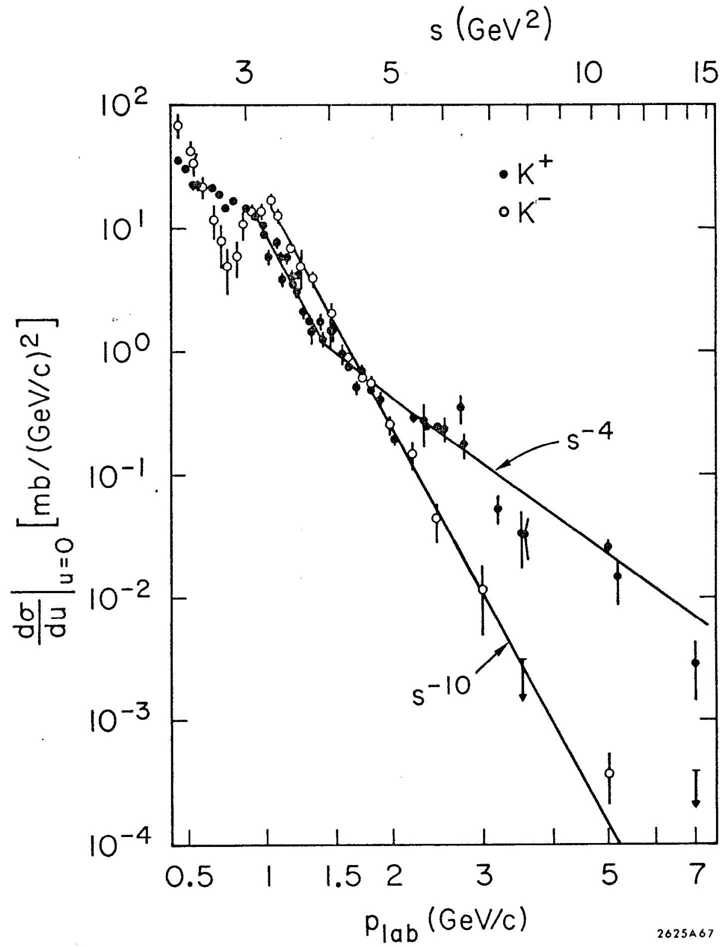
    (b) Find a measured multiplicity distribution in the experimental literature. Does it agree with your prediction?

66. In class, we discussed a dimensional-analysis prediction for the behavior of exclusive cross sections at high Q^2. We obtained the result [Matveev et al., *L. al Nuovo Cimento* **7** (1973) 719]:

$$\frac{d\sigma}{d\Omega}(ab \to cd) \sim s^{3-n_a+n_b+n_c+n_d}, \tag{65}$$

where n_i is the number of constituents (partons) in particle i, assumed to be resolved at the Q^2 scale.

Consider the following picture, showing data for $K^{\pm}p$ backward elastic scattering [Sivers, Brodsky, and Blankenbecler, *Phys. Rept.* **23** (1976) 20]:



Give a qualitative explanation for the behavior of the data in this picture, including any deviations from Eq. 65. You may wish to consider pictures showing x-values for "wee" and non-wee partons in the scattering process.

67. When discussing the January 11 seminar, the question arose concerning how "big" a neutrino detector in a reactor experiment needs to be. You are asked to address this question in this problem.

To be realistic (in terms of the real world flavor of what one is usually confronted with), I'm going to give very little guidance in setting up the problem. However, to make the goal uniform, I'll be specific on a few points: Suppose that you are considering a detector at a 2 km distance from the Diablo Canyon power plant. Imagine that you will take data for three years, and wish to achieve a 1% statistical uncertainty on the measurement of the event rate in your detector. Considering factors such as reactor refueling and detector efficiency, assume that there is an overall detection efficiency of 30%. For simplicity, assume that backgrounds are reduced to a negligible level, and that there is no significant neutrino oscillation. You are asked how big (e.g., how heavy) a detector fiducial volume you need (the actual detector will typically need to be quite a bit larger, in order to deal with issues such as backgrounds).

Don't worry too much about factors of two here, e.g., don't worry about integrating over neutrino energies. In real life, factors of two can be important, but once you have done the basic problem, refinements can in principle be made for a serious proposal – you are still at the "back-of-the-envelope" stage here.

68. We have discussed the parton structure functions, $f(x)$, of a hadron. The level of that discussion was that a deep inelastic scatter resulted in outgoing partons. However, it is hadrons that we eventually observe in an experiment, and a formalism is also needed to describe how a parton evolves into the observed hadrons. A popular framework is that of "fragmentation functions". We define a function, $D_p^h(z)$ to be the probability density for parton p to fragment into a hadron h (plus whatever else) carrying a fraction z of the parton's energy. We have that the total energy of the parton appears in hadrons:

$$\sum_h \int_0^1 z D_p^h(z) dz = 1,$$

and that the number of hadrons h is the sum over all partons fragmenting:

$$\sum_p \int_0^1 D_p^h(z) dz = n_h.$$

The calculation of these functions from first principles is difficult, so people resort to phenomenological models, tuning them with fits to data. From similar arguments as we have seen for the structure functions, we could argue for the form:

$$D_p^h(z) = A\frac{(1-z)^n}{z},$$

where A and n are constants to be obtained from data, or other arguments, and $D_p^h(z) = 0$ below threshold for producing h, of course.

(a) Calculate $\langle z \rangle$, the average energy fraction of hadron h, in terms of A and n.

(b) Show that the average multiplicity of hadron h, produced from a parton of energy Q, grows with Q as

$$n_h \sim \ln(Q/2m_h).$$

Note the similarity with the Feynman scaling result of problem 63. Worth thinking about the connection.