

Physics 231b
Problem Set Number 14
Due Wednesday, February 2, 2005

Note: Some problems may be “review” for some of you. I am deliberately including problems which are potentially in this category. If the material of the problem is already well-known to you, such that doing the problem would not be instructive, just write “been there, done that”, or suitable equivalent, for that problem, and I’ll give you credit.

69. Standard Model Review(?): It is often possible to obtain a QCD result from a QED calculation, by applying a “color factor”. This is possible for graphs which are analogous, and where the color contributions are summed over. The procedure is to take the QED calculation of a cross section, and make the replacement $\alpha^n \rightarrow C_F \alpha_s^n$, where C_F is the “color factor”, sometimes referred to as a “Casimir”, from the fact in group theory that when the elements of a group are averaged over, the result is an operator (Casimir operator) which commutes with each group element.

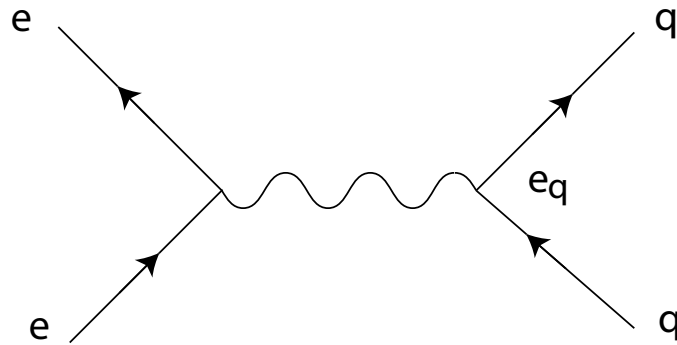
C_F is obtained by writing the QCD graph in terms of color lines, counting all the possible color configurations, and dividing by the number of colors in the initial state. This is similar to the “average over initial polarizations, sum over final polarizations” procedure in calculating unpolarized cross sections. The only subtlety is that an extra factor of $1/2^n$ must be applied, due to historic convention in the definition of the strong coupling constant.

- (a) In deep inelastic ep scattering, we may have contributions from the parton subprocess $\gamma^* q \rightarrow gq$. Show that $C_F = 4/3$, and hence write the cross section for this process given the QED Compton matrix element squared:

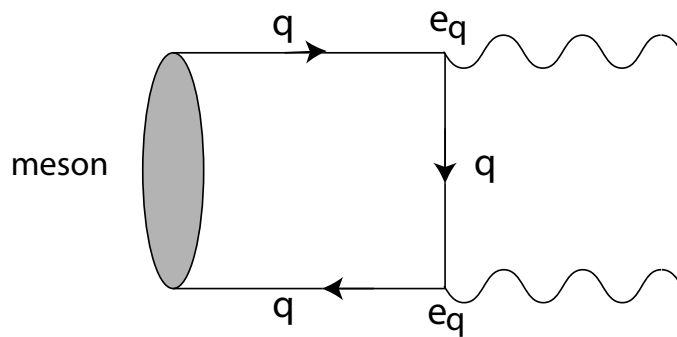
$$\overline{|\mathcal{M}|^2} = 32\pi^2 \alpha^2 \left(-\frac{u}{s} - \frac{s}{u} + \frac{2tQ^2}{su} \right)$$

- (b) What is C_F for the process $\gamma^* g \rightarrow q\bar{q}$? See if you can write down the invariant matrix element squared for this process by crossing symmetry.

70. When we discussed the Han-Nambu quarks in class, we didn't say anything about the gluon electric charges. Correct this deficiency by giving the electric charges of the gluons in the integrally charged quark model (ICQM) of Han and Nambu.
71. We discussed the possibility that quarks "really" are integrally charged, of the Han-Nambu type. We also suggested, in the limit of complete color confinement, that it might not be possible to experimentally distinguish different models which have the same color-averaged charge for a given flavor (and, hence, the issue of which model is "correct" becomes moot). We illustrated this point by considering the cross section for $e^+e^- \rightarrow \text{hadrons}$, which proceeds dominantly (far below the Z^0) via single photon exchange. The amplitude for any graph is thus proportional to the quark charge:

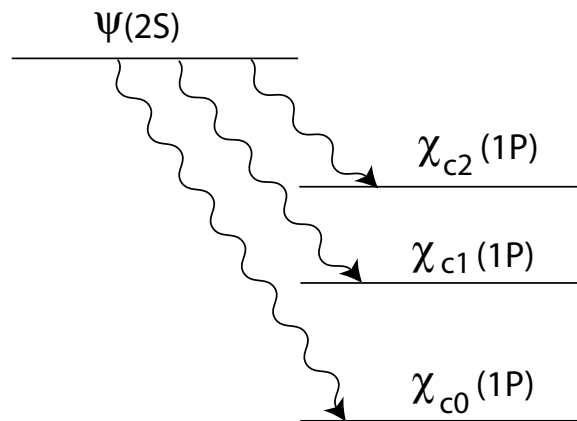


We might get clever, and try to distinguish the models by now considering a process where the amplitude is proportional to the square of the quark charge. For example, consider meson decays to two photons: $M \rightarrow \gamma\gamma$, where we have in mind a perturbation theory graph of the form:



- (a) The amplitude is proportional to e_q^2 , and an average of e_q^2 over color will be different for our two quark models, as you should now demonstrate.
- (b) What is wrong with our clever idea (or does it really work)? In any case, calculate the relative $\gamma\gamma$ decay rate for a meson made of $c\bar{c}$ quarks compared with one made of $b\bar{b}$, both for the integer-charged and the fractional-charged quarks. Assume that meson wave function and phase space factors are identical for the $c\bar{c}$ and $b\bar{b}$ mesons. If you get stuck, a peek at Lipkin, Nucl. Phys. B155, 104 (1979), might be helpful. In your discussion, try to address the graph I have drawn above – what is wrong (or right) with it?

72. We have been discussing the charmonium system in lecture. An important tool in understanding this system, and the potential which binds the c and \bar{c} quarks together, is the radiative transitions which can occur between charmonium states. For example, the first radial excitation of the J/ψ , the $\psi(2S)$ (or ψ'), decays to the lowest P-wave states via the emission of a photon:



This is very analogous to the situation in atomic physics (or, better, positronium), and might be expected to be dominated in this example by an electric dipole transition.

- (a) From what you know about ordinary (non-relativistic) quantum mechanics, predict the relative rates for these transitions:

$$\Gamma(\psi' \rightarrow \gamma\chi_2) : \Gamma(\psi' \rightarrow \gamma\chi_1) : \Gamma(\psi' \rightarrow \gamma\chi_0)$$

[The absolute rate is much harder to predict, as it depends greatly on the actual potential.]

- (b) Compare your prediction with experiment. Of course, you must always include appropriate error analysis in such comparisons.
 - (c) Compare your prediction with experiment, now for the lowest P-wave states of bottomonium.
73. We noted that the J was observed as a resonance in lepton-pair production in hadronic collisions. Let's consider the Drell-Yan model for production of lepton pairs in hadron-hadron collisions. In this model, a parton from hadron 1, with momentum fraction x , collides with a parton from hadron 2, with momentum fraction y , annihilating into an intermediate virtual photon, which then yields an $\ell^+\ell^-$ pair. Let the invariant mass of the $\ell^+\ell^-$ pair be Q^2 , and let s be the overall center-of-mass energy squared for the process $h_1h_2 \rightarrow \ell^+\ell^-X$.

Consider π^\pm scattering on carbon-12 at high energy. Predict the ratio:

$$\frac{\sigma(\pi^+C \rightarrow \ell^+\ell^-X)}{\sigma(\pi^-C \rightarrow \ell^+\ell^-X)}$$

in the two kinematic regimes $Q^2/s \approx 0$ and $Q^2/s \approx 1$.