Physics 231b
Problem Set Number 15
Due Wednesday, February 9, 2005

Next quarter Michael Ramsey-Musolf will be teaching Ph 231. He will emphasize the calculational aspects of the Standard Model, particularly in the electroweak area. Anticipating this, I'll do things a bit differently than usual in Ph 231b, so as to complement rather than duplicate his material. Here is the course description for third quarter:

Ph231c
Title: Field Theory and the Standard Model
Prerequisites: Ph205ab
Description: Applications of quantum field theory to the Standard Model, focusing on the electroweak interaction. The course will include a brief review of the Weinberg-Salaam model and its renormalization, followed by applications to weak decays, Z-pole tests, W and Higgs boson studies, low-energy neutral current phenomena, CP- and T-violation, and neutrino properties. Emphasis will be on applications with direct relevance to electroweak phenomenology.
Instructor: Ramsey-Musolf
MW 2:30-4:00

74. In class, we looked at various decay widths for charmonium → hadrons according to lowest order QCD perturbation theory. Let us see how well some of these predictions compare with experiment: The following data appear in the 2004 edition of the "Review of Particle Properties":

$$\Gamma_{TOT}(\eta_c) = 17.3^{+2.7}_{-2.5}\ \mathrm{MeV}$$

$$\Gamma_{TOT}(J/\psi) = 91.0 \pm 3.2\ \mathrm{keV}$$

$$\Gamma_{e^+e^-}(J/\psi) = 5.40 \pm 0.15 \pm 0.07\ \mathrm{keV}$$

$$\Gamma_{TOT}(\chi_{c0}) = 10.1 \pm 0.8\ \mathrm{MeV}$$

$$\Gamma_{TOT}(\chi_{c2}) = 2.11 \pm 0.16\ \mathrm{MeV}$$

(a) Compare these results with appropriate ratios of the following lowest order predictions:

$$\Gamma(^3S_1 \to e^+e^-) = \frac{4\alpha^2 e_q^2}{M^2} \mid \psi(0) \mid^2$$

$$\Gamma(^3S_1 \to \text{hadrons}) = \frac{40}{81\pi}(\pi^2 - 9)\alpha_s^3 \frac{| \psi(0) |^2}{M^2}$$
$$\Gamma(^1S_0 \to \text{hadrons}) = \frac{8}{3}\alpha_s^2 \frac{| \psi(0) |^2}{M^2}$$
$$\Gamma(^3P_0 \to \text{hadrons}) = 96\alpha_s^2 \frac{| \psi'(0) |^2}{M^4}$$
$$\Gamma(^3P_2 \to \text{hadrons}) = \frac{128}{5}\alpha_s^2 \frac{| \psi'(0) |^2}{M^4}$$

[If you are careful, you will pay attention to such points as the fact that $\Gamma_{TOT}(J/\psi) \neq \Gamma_{\text{hadrons}}(J/\psi)$]. The errors aren't too badly correlated among these measurements, so you can use simple uncorrelated error propagation. Comment on how well or poorly things fit. Note that I want you do do a defensible job of the error analysis and consistency test. If the fit isn't perfect, what might be the problem?

(b) Using what is known about the bottomonium system, make a prediction for the width of the η_b. Since this state has not been found yet, you'll have to wait to find out how well you did.

75. In problem 63, we introduced the notion of Feynman scaling, with the consequence that the mean multiplicity (number of particles produced) is expected to grow like $A \ln s$ for large $s = E_{cm}^2$. In problem 65, you created a simple model for the multiplicity distribution. Let us carry the discussion further:

(a) Show that, as a consequence of Feynman scaling, the k^{th} moment of the multiplicity distribution ("multiplicity distribution"≡ probability distribution for number of produced particles) is:

$$\langle n^k \rangle = A_k (\ln s)^k + O[(\ln s)^{k-1}]$$

Hints: First, note that, in the large $\ln s$ limit, and assuming we have integrated over all d^2p_T, the one particle inclusive cross section (σ_1) scales according to:

$$\frac{1}{\sigma_T}\frac{d\sigma_1}{dy} = f_1 = \text{constant}$$

(σ_T is the total cross section)

Of course: (as you were supposed to show in problem 59)

$$\langle n \rangle = \int_{y_{min}}^{y_{max}} \frac{1}{\sigma_T} \frac{d\sigma_1}{dy} dy = f_1 \times (y_{max} - y_{min}) \sim \ln s$$

We assume that emission of particles is uncorrelated in the large energy limit, so the two particle inclusive cross section scales also:

$$\frac{1}{\sigma_t} \frac{d^2\sigma_2}{dy_1 dy_2} = f_2 = \text{constant},$$

and, in general, the m particle inclusive cross section ($A + B \rightarrow 1 + 2 + ... + m + X$):

$$\frac{1}{\sigma_T} \frac{d^m\sigma_m}{dy_1 ... dy_m} = f_m = \text{constant}.$$

Once you convince yourself that:

$$\langle n(n-1)...(n-k+1) \rangle = \overbrace{\underbrace{\int \cdots \int}_{\{y_{min}\}}}^{\{y_{max}\}} \frac{1}{\sigma_T} \frac{d^k\sigma_k}{dy_1 ... dy_k} dy_1 ... dy_k,$$

you can quickly proceed to the desired result.

(b) Thus, show that the multiplicity distribution, $P_n(s) \equiv$ probability to produce n particles, at given $E_{cm}^2 = s$, with normalization:

$$\sum_{n=0}^{\infty} P_n(s) = 1$$

has the simple scaling form:

$$P_n(s) = \frac{1}{\langle n \rangle} \psi \left(\frac{n}{\langle n \rangle} \right)$$

where ψ is a function of $z = \frac{n}{\langle n \rangle}$ only, and the s dependence enters only in $\langle n \rangle = \langle n \rangle (s)$. Hint: Write down $\sum_n P_n(s) n^k \cong \langle n^k \rangle$ and change the $\sum$ to an $\int$ in the large s limit.

Discussion: This result, which works amazingly well even at quite low energies, is called KNO scaling after Kobo, Nielsen, Olesen, Nucl. Phys. **B40** 317 (1972). There are a number of questions of rigor in the above argument, which you most likely have glossed over. See the above reference for more thourough attention to such matters.

76. We discussed the "13% rule," in which we expect (according to perturbation theory, in the non-relativistic heavy quarkonium model):

$$\frac{B(\psi' \to \text{hadrons})}{B(\psi \to \text{hadrons})} = \frac{B(\psi' \to e^+e^-)}{B(\psi \to e^+e^-)} = (12.7 \pm 0.6)\%$$

   (a) Check whether this rule is satisfied. When determining the measured inclusive branching ratios, it is all right to include all processes (including, perhaps, radiative ones) for which the derivation of the above rule should be valid, but be sure to exclude other decay modes.

   (b) Now make the corresponding test in the $\Upsilon(1S)$–$\Upsilon(2S)$ system.

77. Consider the production of the $\Upsilon(1S)$ in e^+e^- collisions, and its decay into $\Lambda\bar{\Lambda}$.

   (a) In the center-of-mass frame, what is the most general angular distribution that the Λ can have? Choose the z-axis to be along the positron direction.

   (b) According to the QCD helicity theorem, what angular distribution do you expect?