Physics 231b
Problem Set Number 16
Due Wednesday, February 16, 2005

78. Consider the decay $\tau^- \to \nu_\tau e^- \bar{\nu}_e$.

    (a) Calculate the decay rate you expect (in lowest order) for this process.

    (b) Make a simple prediction for the branching fraction for $\tau^- \to \nu_\tau e^- \bar{\nu}_e$. Compare with experiment.

    (c) Using the measured branching fraction, and your calculated decay rate for $\tau^- \to \nu_\tau e^- \bar{\nu}_e$, predict the total lifetime of the τ lepton. Compare with experiment.

79. One of the promising ideas for going beyond the standard model is the idea of supersymmetry, a symmetry between bosons and fermions. Let us do some bookkeeping in this context. If there is a symmetry between bosons and fermions, then we should have the same number of degrees of freedom (fields) in bosons as in fermions: To each fermionic degree of freedom in our "particle" world there should be a corresponding bosonic degree of freedom in the supersymmetric "sparticle" world. For example, the Dirac electron (e) has four degrees of freedom. The corresponding "selectrons" ($\tilde{e}$) come as two complex scalar fields, for a total of four degrees of freedom. The weak eigenstates of the selectrons are sometimes denoted $\tilde{e}_L$ and $\tilde{e}_R$, where the subscripts denote partnership with left- and right-handed coupling electrons. The mass eigenstates could be mixtures of these. According to the symmetry, the couplings are the same as for particles, in particular, the electric charges of the two $\tilde{e}_L$ degrees of freedom are ± 1, if the positron charge is $+1$.

    The partner to the photon is called the "photino", and is a spin-1/2 field with two degrees of freedom (think of a Majorana neutrino). The situation is complicated slightly because: (i) In the minimal supersymmetric extension to the standard model two complex scalar higgs fields are required (why?). As before, three of the degrees of freedom are eaten up by the longitudinal degrees of freedom of the weak bosons. (ii) Mixing can occur among the various "chargino" ($\tilde{\chi}_i^\pm$) sparticles (partners of the $W^{\pm}$ and charged higgses) and also among the "neutralinos" ($\tilde{\chi}_i^0$) (partners of the photon, Z, neutral higgses).

(a) Start the bookkeeping by first listing all of the fundamental particle fields (after symmetry breaking) in this minimal extension to the standard model.

(b) Now list all of the fundamental sparticle partners to the fields in part (a). You may classify them either according to weak eigenstate or mass eigenstate (the $\tilde{\chi}$ symbol is typically used for the mass eigenstates), but make sure you get the same number of degrees of freedom.

80. We have discussed the CVC ("Conserved Vector Current") hypothesis for the weak interaction. Consider the decay of the Σ baryon to a Λ baryon. The most general form of the relevant hadronic weak vector current is:

$$V^\mu = \bar{u}(p_\Lambda)\left[f_1(q^2)\gamma^\mu + if_2(q^2)\sigma^{\mu\nu}q_\nu + f_3(q^2)q^\mu\right]u(p_\Sigma). \qquad (66)$$

(a) Assuming CVC, find any relationships among the form factors. What can you say about the form factors at the $q^2 = 0$ kinematic limit?

(b) Calculate the $\Sigma \to \Lambda\gamma$ decay rate. The electromagnetic current is, as you know, conserved. From the measured rate and masses, determine the value of the form factor that should appear in your expression. Note that this is a measure of the transition magnetic moment.

81. Some Z phenomenology:

(a) Calculate the rate $\Gamma(Z \to f\bar{f})$ for the Z to decay to a fundamental fermion (lepton or quark) and its antifermion. Assume the standard model, and calculate the lowest order only. You may neglect the fermion mass if you wish. Express your answer in terms of the vector and axial vector coupling constants

$$\begin{aligned} c_V^f &= T_f^3 - 2Q_f \sin^2\theta_W \\ c_A^f &= T_f^3. \end{aligned}$$

Do the axial and vector contributions interfere? Explain in simple physical arguments why or why not.

(b) Using your result of part (a), calculate the total Z width you expect.

Compare with experimental results.