Physics 231b
Problem Set Number 17
Due Wednesday, February 23, 2005

82. We have been discussing electromagnetic showers. Let us here think about the very important issue of energy resolution. There are several contributions to the energy resolution of an electromagnetic calorimeter, which are adjusted in the design process of optimizing the detector:

(a) Shower fluctuations: Even in a "total absorption" (as opposed to "sampling") calorimeter, the shower is not fully contained in the device. Based on our discussion in class, estimate the average energy leakage, as a function of incident electron energy, for a detector that is 16 X_0 thick. The actual leakage will of course fluctuate about this value. Assuming that the (rms) fluctuation is the same as the average leakage, give the energy resolution as a function of energy due to this source. Note that we are only including leakage out the back of the calorimeter; we are here assuming that we have full containment in the transverse directions. In practice, this is also a concern, and the leakage contribution to energy resoluton is studied with Monte Carlo simulations and with test beams of known energy.

To do this computation, note that the very simple model we developed is really inadequate (what happens?), though we could imagine refinements. Instead, I suggest you use the empirical formula based on other studies, which I'll give in simplified form here. The average number of $e^\pm$ with $E > 2$ MeV crossing a plane at a depth x into the material is:

$$N \approx N_0 \left(\frac{x}{X_0}\right)^a e^{-bx/X_0}, \tag{67}$$

where

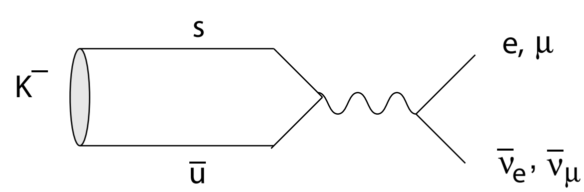
$$\begin{aligned} N_0 &= 5.5E_0(\mathrm{GeV})\sqrt{Z}b^{a+1}/\Gamma(a+1) \\ a &= 2 + 0.6\ln E_0 \\ b &= 0.6. \end{aligned} \tag{68}$$

Knowing the scale X_0 is not quite sufficient; you may take $Z = 54$, as roughly appropriate for CsI. Finally, you need some simple model for the energy distribution of these N particles with $E > 2$ MeV. I suggest trying something like a $1/E$ spectrum up to a cut-off at 10 MeV (approximate critical energy), and perhaps extrapolating this down to $1/2$ MeV, at least to see how much difference this makes.

(b) Make a graph of fractional energy resolution vs incident electron energy based on the above, plus the following additional contributions: (*i*) A typical calorimeter has many channels, and it is not possible to get the relative calibrations exactly correct. This introduces an additional source of effective fluctuations when including showers in different parts of the detector, called "inter-calibration error". Assume that the (rms) spread in calibration uncertainty is 0.5%. (*ii*) Electronic noise introduces another contribution to the resolution, typically corresponding to an energy fluctuation that is approximately independent of incident energy. Assume that the electronic noise introduces a 1 MeV (rms) contribution to the measurement of a shower energy. (*iii*) Background from other tracks and accelerator "spray" can overlap with the desired shower and introduce another source of fluctuation in the energy measurement. We'll neglect this here.

(c) For your detector, what is the mass resolution for a $H \to \gamma\gamma$ decay, for a Higgs mass of 120 GeV, and a Higgs momentum of 200 GeV? State any further assumptions you make. Try to find the expected performance of the CMS LHC detector and compare.

83. Consider the leptonic decays of the K^-: $K^- \to e^-\bar{\nu}_e$ and $K^- \to \mu^-\bar{\nu}_\mu$. The amplitudes (in lowest order) for these processes must be:



$$\mathcal{M} = \frac{G}{\sqrt{2}} J_K^\mu \bar{u}(p)\gamma_\mu(1-\gamma^5)v(k).$$

The hadronic current, J_K^μ, is not readily calculated, due to the fact that the quarks are bound in the kaon. However, it must be a Lorentz 4-vector, and, since the kaon is spinless, can only be of the form:

$$J_K^\mu = f(q^2)V_{us}q^\mu = f_K V_{us} q^\mu,$$

where we define $f_K = f(m_K^2)$, since $q^2 = m_K^2$.

(a) Calculate a formula for the total decay rate for these processes.

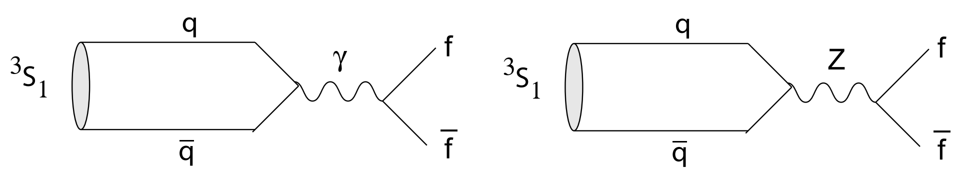
(b) What do you expect for the ratio:

$$\frac{\Gamma(K^- \to e^-\bar{\nu}_e)}{\Gamma(K^- \to \mu^-\bar{\nu}_\mu)}?$$

Does this agree with your qualitative expectation (explain)? Does it agree with experiment?

84. Consider the decays $D^0 \to K^-\pi^+$, $D^0 \to \pi^-\pi^+$, and $D^0 \to K^+\pi^-$. What do you expect for the relative rates for these decays? Evaluate numerically your expected ratios of the rates, and compare your prediction with experiment.

85. We consider the decay of a quarkonium resonance into a fermion-antifermion pair. In particular, consider the decay of a vector resonance, via the graphs:



These graphs may be readily calculated, giving the result:

$$\Gamma(^3S_1 \to f\bar{f}) = c_f \frac{\Gamma_0}{e_Q^2}\left[\left|e_Q e_f + \frac{1}{y^2}\frac{v_Q v_f M_V^2}{M_V^2 - M_Z^2 + iM_Z\Gamma_Z}\right|^2 + \left|\frac{1}{y^2}\frac{v_Q a_f M_V^2}{M_V^2 - M_Z^2 + iM_Z\Gamma_Z}\right|^2\right],$$

where M_V is the $^3S_1$ mass, $y = 2\sin 2\theta_W$, $v_f = 2I_{3f} - 4e_f \sin^2\theta_W$, $a_f = 2I_{3f}$, c_f is the number of colors that fermion f comes in, and $\Gamma_0 \equiv \Gamma(^3S_1 \xrightarrow{\gamma} e^+e^-) = 16\pi\alpha^2 e_Q^2 \frac{|\psi(0)|^2}{M_V^2}$. You should convince yourself that you could derive this formula if I hadn't given you the answer.

(a) Calculate the branching ratio in the standard model for $\Upsilon(1S) \rightarrow \nu\bar{\nu}$, and for $J/\psi \rightarrow \nu\bar{\nu}$.

(b) Using the decay $\Upsilon(2S) \rightarrow \pi\pi\Upsilon(1S)$ to tag $\Upsilon(1S)$ decays, determine the integrated luminosity required at an e^+e^- collider in order to measure the number of light neutrino flavors at the 3σ level. You may assume an efficiency of 30%, and, possibly unrealistically, no background.