Physics 231b
Problem Set Number 18
Due Wednesday, March 2, 2005

86. Some top quark phenomenology: The 2004 RPP world average value for the top quark mass is 174.3 ± 5.1 GeV, from D0 and CDF "direct" mass measurements (although this value is already modified somewhat by more recent results).

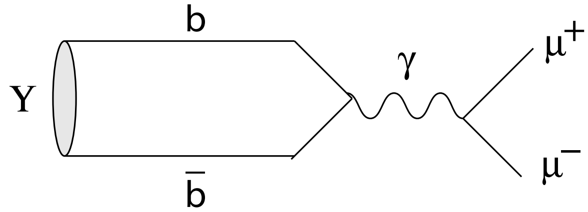
    (a) Predict the total decay rate of the top quark, in lowest order perturbation theory, in the 3 generation standard model. As always, you should state any assumptions or approximations used, but at least do it sufficiently accurately that you expect the current experimental uncertainty in the mass to be the dominant uncertainty in your prediction.

    (b) Do you expect large corrections to your prediction of part (a) from the strong interaction? Justify your answer.

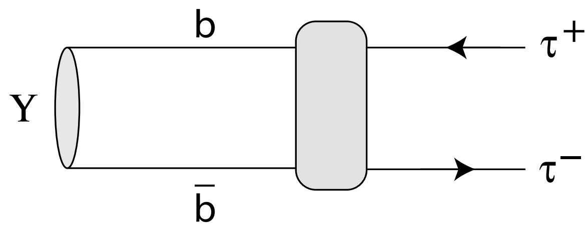
    (c) Predict the splitting between the lowest pseudoscalar T and vector T^* mesons, consisting of a t quark and a $\bar{u}$ or $\bar{d}$ antiquark.

87. The possibility that quarks and leptons are composite objects is an interesting potential modification to the standard model. Various limits exist on the scale of such compositeness, tending to be most stringent for the first and second generation fermions. It has been argued that compositeness may be more significant in the third generation. In particular, Pati and Stremnitzer (J. C. Pati and H. Stremnitzer, *Phys. Rev. Lett.* **56** (1986) 2152), suggested that large effects might be observable in toponium decays. This was before the top quark mass was known to be so large (and hence the idea is impractical – why?), but we may be able to probe the same possibility with Upsilon decays.

    The basic idea is that processes relating third generation fermions may have additional contributions, due to compositeness, compared with processes relating third with first or second generation fermions. Thus, in Upsilon decays to $\mu^+\mu^-$ we have the lowest-order graph (another graph with Z exchange also contributes, but is small):



For decays to $\tau^+\tau^-$, we have the corresponding graph, but according to this idea of compositeness, we also have:



The gray box is supposed to suggest some sort of rearrangement of shared constituents among the quarks and leptons. Let us define:

$$R \equiv \frac{\Gamma(\Upsilon(1S) \to \tau^+\tau^-)}{\Gamma(\Upsilon(1S) \to \mu^+\mu^-)}.$$

(a) Give your prediction for R in the standard model (that is, without the additional graph) and compare with experiment. Lowest-order perturbation theory is sufficient, but calculate it so that the prediction should be good to better than 1/2%.

Pati and Stremnitzer's argument, when applied to the $\Upsilon(1S)$ suggest that deviations from the standard model prediction for R could be fairly large, and that measurements at the level of 1% are certainly interesting. Let us investigate what it would take to make a 1% measurement of R in e^+e^- collisions. We might consider three approaches:

i. Run on the $\Upsilon(1S)$, and count the number of $\tau^+\tau^-$ and $\mu^+\mu^-$ events.

ii. Run on the $\Upsilon(1S)$, as above, but also in the continuum just below the $\Upsilon(1S)$.

iii. Run on the $\Upsilon(2S)$, and "tag" $\Upsilon(1S)$ decays via the $\Upsilon(2S) \to \pi\pi\Upsilon(1S)$ decays.

(b) Comment on the relative merits of these different approaches. Which method do you expect to yield ultimately the best accuracy?

(c) Pick your "best accuracy" method, and estimate the integrated luminosity required to make a $1/2\%$ measurement of R.