

Physics 231b
Problem Set Number 19
Due Wednesday, March 9, 2005

88. We have been discussing the measurements of the CKM matrix elements. We have made a number of checks for consistency with the standard model along the way. You are here asked to make another check.

(a) In particular, we have the measurements:

$$|V_{ud}| = 0.09738 \pm 0.0005 \quad (69)$$

$$|V_{us}| = 0.2200 \pm 0.0026. \quad (70)$$

Use these measurements, and the unitarity of V as a three-dimensional matrix, to predict $|V_{ub}|$ in the standard model.

(b) The measured value of $|V_{ub}|$ is 0.00367 ± 0.00047 . Compare your prediction with this measurement. Are they consistent? Note that a “ 3σ ” deviation is already unlikely, and a “ 4σ ” deviation would be considered by most people as an exciting indication that the standard model is broken. Do you see an effect of this magnitude? If so, think carefully about what you have done, and if you find a problem see whether you can come up with a better check for consistency with the standard model.

89. A tachyon is a particle that travels faster than the speed of light (*i.e.*, is “superluminal”). No evidence has been found to support their existence, but the concept can be useful. You are invited to pursue the group theory of Poincare transformations into the extended regime including superluminal transformations. We’ll try a couple of simple ideas here.

(a) Let us extend the notion of invariance of the scalar product slightly by relating 4-vector quantities in two frames (primed and unprimed) according to transformations with the property that:

$$x'^{\mu}x'_{\mu} = \pm x^{\mu}x_{\mu}.$$

The plus sign corresponds to the subluminal transformations that we are used to, and the minus sign corresponds to superluminal transformations. The relation between energy and momentum in a subluminal frame is $E^2 - \mathbf{p}^2 = m^2$. What is the relation in a superluminal frame?

- (b) The relation between spacetime coordinates under a subluminal transformation given by a Lorentz boost of speed u along the z axis may be written:

$$\begin{aligned} t' &= \frac{t - uz}{\sqrt{1 - u^2}} \\ x' &= x \\ y' &= y \\ z' &= \frac{z - ut}{\sqrt{1 - u^2}}. \end{aligned}$$

Up to some possible sign uncertainties, see if you can give the corresponding rule for a superluminal transformation, by speed $v > 1$ along the z axis.

- (c) Assume that tachyons may be treated as point particles in the usual sense, and that they have “ordinary” electromagnetic interactions with normal (bradyonic, or subluminal) matter. For example, suppose there exist tachyons which can have localized trajectories, and which appear to carry an electric charge equal to that of the positron. Devise an experiment to detect such particles. You may suppose them to have an astronomical origin if you wish. Note that we are making some pretty strong and potentially incorrect assumptions here.

90. We have been discussing mixing and CP violation in the neutral B meson system. The mass eigenstates are related to the flavor eigenstates according to:

$$\begin{aligned} |B_L\rangle &= p|B^0\rangle + q|\bar{B}^0\rangle \\ |B_H\rangle &= p|B^0\rangle - q|\bar{B}^0\rangle. \end{aligned}$$

The mass and decay matrix in the flavor basis is

$$H = \begin{pmatrix} M - \frac{1}{2}i\Gamma & M_{12}^* - \frac{1}{2}i\Gamma_{12}^* \\ M_{12} - \frac{1}{2}i\Gamma_{12} & M - \frac{1}{2}i\Gamma \end{pmatrix},$$

and parameters M_{12} and Γ_{12} are calculated from the mixing box diagrams, as we discussed in class.

- (a) To the dominant leading non-zero terms in the quark masses for M_{12} and Γ_{12} , show that $|p/q|=1$, a result we have used in our discussion in class.
- (b) What is the condition on M_{12} and Γ_{12} such that $[H, CP] = 0$? Is this condition met according to the leading contributions of part (a)?

91. In problem 83, you considered leptonic kaon decays.

- (a) Using your result for $K \rightarrow \mu\nu$, and numbers from RPP, calculate f_K .
- (b) Do the same for f_π , f_D , f_{D_s} , and f_B , and compare.