Ph196bEM Homework1 Due: Friday, Feb. 20, 5pm, 337 Lau.

1.Jackson (3rd Edition), chapter 1, problem 1.1.

2. Lines of Force.

Electric fields are often represented by maps of their lines of force.

A line of force is defined by a curve $\vec{r}(s)$ satisfying, at each point where $E = |\vec{E}|$ is finite and non-zero, $\frac{d r_i}{d s} = \frac{E_i}{E}$. A point where $E = 0$ is called a neutral point and there is no unique line of force through it; similarly, at a point charge q, there is not a unique line of force. You can think of lines of force beginning or ending at these special points.

Further a tube of flux in a charge-free region is defined by an open surface $\mathcal{S}$ with perimeter $\mathcal{P}$ containing no neutral points or point charges and the set of all lines of force through the points of $\mathcal{P}$ with no line of force passing through more than one point of $\mathcal{P}$. The flux in the tube is defined by $\mathcal{F} = \int_{\mathcal{S}} \vec{E} \cdot d\vec{A}$ with $d\vec{A}$ defined over $\mathcal{S}$ with one sense or the other; it is easy to see that the flux through any cross section $\mathcal{S}'$ of the tube is just $\mathcal{F}$ as no flux passes into or out of the tube through the wall defined by the lines of force.

In geometrically symmetric situations, tubes of flux are often easy to calculate and may be used to simplify the calculation of lines of force, thereby avoiding the differential equation.

As a simple example, a cone of half-angle θ with vertex on a point charge q carries a flux of $\frac{q}{2\varepsilon_0}$ (1-cos θ) since the lines of force are radial from the charge. You can think of the surface $\mathcal{S}$ as a spherical cap over the end of a truncated cone centrated at the origin having a radius ϵ of arbitrarily small value.

Suppose $q_0 = 1$ is located at the origin and $q_1 = 2$ is located at $\vec{r}_1 = (0, 0, -1)$. Let a line of force make an angle θ with the z-axis as it approaches the origin. Find the angle ψ that this line makes with the $+z$-axis as it recedes to infinity.

3.Jackson, Ch. 1 problem 1.4.

4.Jackson, Ch. 1 problem 1.12.

5.Jackson, Ch. 2 problem 2.1, subsectons a, b, c, d, f.

6.Jackson, Ch. 2 problem 2.2.