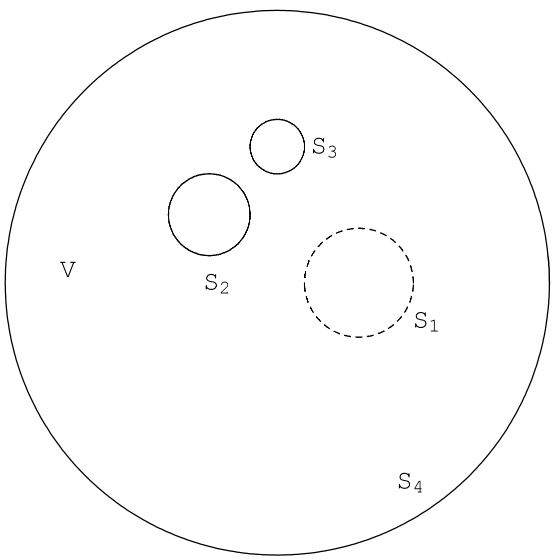
Ph196bEM Homework3 Due: Friday, Mar 5, 5pm, 337 Lauritsen

12. Jackson problem 1.7

13. Jackson problem 1.17.

Note that the bounding surface may be disjoint so this problem covers the case of several separated lumps of conducting stuff; further, be sure to notice that there are no charges in the problem except those on the conductors. Think of the geometry like this



14. Jackson problem 1.18.

Note the typo in part a): the integral should be

$\int_{S_1} \ldots\, da'$ rather than $\int_{S_1} \ldots\, d^3x'$, i.e., it is a surface integral.

Also you may find it clarifying to introduce a potential other than unity for surface S_1, and let $\sigma_1(\vec{r} \in S_1)$ be the charge density on S_1 when it is at potential ϕ_1 and carries a total charge $Q_1 = \int_{S_1} \sigma_1 \, dA$ when all other parts of the boundary of V are grounded.

Be sure to notice that, for the Green function described in the problem, $\frac{1}{4\pi\varepsilon_0} \int_{S_1} \sigma_1(\vec{r}\,')\, G(\vec{r} \in S_1; \vec{r}\,')\, dA' = \phi_1$.

15. Jackson, problem 1.20.

16. A Capacitance Matrix

A large circuit board (30 cm by 30 cm) is covered with a thin layer of copper to act as a ground plane (i.e., the copper is connected to the laboratory's ground system and is at zero potential). For mechanical convenience, the copper is bonded to a thin insulating plastic board for support. Assume that no components are mounted on the circuit board so the ground plane is "clean". Near its center are three circular holes, each of diameter 2mm, fully etched through the copper. The centers of the three holes are on the vertices of an equilateral triangle with 0.5 cm sides. Centered in each of the holes is a circular copper pad of diameter 1 mm, well insulated from the copper ground plane.

Of course, it's too hard to analytically calculate electrostatics problems relating to the pads in this geometry (except numerically and that's a lot of work), but the only electrical information that will be needed in the final application of the circuit board are various capacitances. Unfortunately, someone has ripped off your handy capacitance meter, but, rummaging around the lab, you find a fancy electrostatic voltmeter, a 100 volt power supply, and a precision integrating ammeter, i.e., an instrument that can tell you the integral of the current that has flowed through it. Of course, you know that an electrostatic voltmeter measures the voltage between two terminals without drawing any current from them . Suppose you can assume that the voltmeter's capacitance to ground is negligible compared to any other capacitance in the problem. With the negative lead from the power supply always connected to ground, you do the following sequence of operations and measurements. Assume in all of the following that the wires used in hooking things up are thin enough and are laid out carefully enough to have negligible effects. You may find it useful to make a circuit theory sketch illustrating the steps taken.

i) First, the electrostatics system is reset, i.e., the three pads are connected together and then to ground. The wires are then removed.

ii) One of the pads is then reconnected to ground and one of the others is connected to the positive lead of the 100 volt power supply through the integrating ammeter. Care is taken to leave the third pad undisturbed. The integrator shows that a charge of 1 nanoCoulomb (10^{-9} C) flows to the pad as it charges to 100 V above ground.

iii) The connections to the pads are removed, and, as at the beginning, the system is reset, and then all the wires are removed.

iv) Lastly, the positive lead of the 100 V power supply is connected to one of the pads. The voltage of one of the other two pads is then measured with the electrostatic voltmeter and found to be 25 V above ground.

a) From the given data write the capacitance matrix for the geometry in units of pF (picoFarads).

b) If you leave the power supply and voltmeter connected as in item iv) above, and then touch the power supply's positive lead to the third pad, what will the voltmeter read (give the numerical answer in volts)?

c) Finally, you carefully disconnect the power supply from the two pads and replace the voltmeter with a 10 Ohm resistor to ground. How much energy (give the result numerically in nJ, nanoJoules) is dissipated in the resistor by the current that flows through it?

Notice that the electrostatic field is negligible near the edges of the ground plane so you can consider them to be at infinity. Thus the geometry has three-fold rotational symmetry. Exploit it in writing the capacitance matrix.