Fig. 32.1: Stopping power \( (= \langle -dE/dx \rangle) \) for positive muons in copper as a function of \( \beta\gamma = p/Mc \) over nine orders of magnitude in momentum (12 orders of magnitude in kinetic energy). Solid curves indicate the total stopping power. Data below the break at \( \beta\gamma \approx 0.1 \) are taken from ICRU 49 [4], and data at higher energies are from Ref. 5. Vertical bands indicate boundaries between different approximations discussed in the text. The short dotted lines labeled “\( \mu^- \)” illustrate the “Barkas effect,” the dependence of stopping power on projectile charge at very low energies [6]. \( dE/dx \) in the radiative region is not simply a function of \( \beta \).
Bethe curve
(closely related: the Bethe-Bloch curve)

Figure 32.2: Mean energy loss rate in liquid (bubble chamber) hydrogen, gaseous helium, carbon, aluminum, iron, tin, and lead. Radiative effects, relevant for muons and pions, are not included. These become significant for muons in iron for $\beta\gamma \gtrsim 1000$, and at lower momenta for muons in higher-$Z$ absorbers. See Fig. 32.23.
**Figure 32.8:** Straggling functions in silicon for 500 MeV pions, normalized to unity at the most probable value $\delta_p/x$. The width $w$ is the full width at half maximum.
Multiple Coulomb scattering

**Figure 32.10:** Quantities used to describe multiple Coulomb scattering. The particle is incident in the plane of the figure.

\[
\theta_0 = \frac{13.6 \text{ MeV}}{\beta cp} \approx \sqrt{x/X_0} \left[ 1 + 0.038 \ln(x/X_0) \right]
\]
32.4.2. **Radiation length**: High-energy electrons predominantly lose energy in matter by bremsstrahlung, and high-energy photons by $e^+e^-$ pair production. The characteristic amount of matter traversed for these related interactions is called the radiation length $X_0$, usually measured in g cm$^{-2}$. It is both (a) the mean distance over which a high-energy electron loses all but $1/e$ of its energy by bremsstrahlung, and (b) $7/9$ of the mean free path for pair production by a high-energy photon [42]. It is also the appropriate scale length for describing high-energy electromagnetic cascades. $X_0$ has been calculated and tabulated by Y.S. Tsai [43]:

$$\frac{1}{X_0} = 4\alpha r_e^2 \frac{N_A}{A} \left\{ Z^2 [L_{\text{rad}} - f(Z)] + Z L'_{\text{rad}} \right\}. \quad (32.26)$$

For $A = 1$ g mol$^{-1}$, $4\alpha r_e^2 N_A/A = (716.408 \text{ g cm}^{-2})^{-1}$. $L_{\text{rad}}$ and $L'_{\text{rad}}$ are given in Table 32.2. The function $f(Z)$ is an infinite sum, but for elements up to uranium can be represented to 4-place accuracy by

$$f(Z) = a^2 \left[ (1 + a^2)^{-1} + 0.20206 - 0.0369 a^2 + 0.0083 a^4 - 0.002 a^6 \right], \quad (32.27)$$

where $a = \alpha Z$ [44].
Figure 32.11: Fractional energy loss per radiation length in lead as a function of electron or positron energy. Electron (positron) scattering is considered as ionization when the energy loss per collision is below 0.255 MeV, and as Möller (Bhabha) scattering when it is above. Adapted from Fig. 3.2 from Messel and Crawford, Electron-Photon Shower Distribution Function Tables for Lead, Copper, and Air Absorbers, Pergamon Press, 1970. Messel and Crawford use $X_0$(Pb) = 5.82 g/cm$^2$, but we have modified the figures to reflect the value given in the Table of Atomic and Nuclear Properties of Materials ($X_0$(Pb) = 6.37 g/cm$^2$).
Figure 32.15: Photon total cross sections as a function of energy in carbon and lead, showing the contributions of different processes [51]:

\[ \sigma_{\text{p.e.}} = \text{Atomic photoelectric effect (electron ejection, photon absorption)} \]

\[ \sigma_{\text{Rayleigh}} = \text{Rayleigh (coherent) scattering—atom neither ionized nor excited} \]

\[ \sigma_{\text{Compton}} = \text{Incoherent scattering (Compton scattering off an electron)} \]

\[ \kappa_{\text{nuc}} = \text{Pair production, nuclear field} \]

\[ \kappa_{\text{e}} = \text{Pair production, electron field} \]

\[ \sigma_{\text{g.d.r.}} = \text{Photonuclear interactions, most notably the Giant Dipole Resonance [52].} \]

In these interactions, the target nucleus is broken up.

Original figures through the courtesy of John H. Hubbell (NIST).
Electromagnetic showers
Electromagnetic showers

Figure 32.20: An EGS4 simulation of a 30 GeV electron-induced cascade in iron. The histogram shows fractional energy deposition per radiation length, and the curve is a gamma-function fit to the distribution. Circles indicate the number of electrons with total energy greater than 1.5 MeV crossing planes at $X_0/2$ intervals (scale on right) and the squares the number of photons with $E \geq 1.5$ MeV crossing the planes (scaled down to have same area as the electron distribution).
~GeV muons and electrons in a hydrocarbon detector
Hadronic showers
Cherenkov radiation

32.7.1. **Optical Cherenkov radiation**: The angle $\theta_c$ of Cherenkov radiation, relative to the particle's direction, for a particle with velocity $\beta c$ in a medium with index of refraction $n$ is

$$\cos \theta_c = \frac{1}{n \beta}$$

or

$$\tan \theta_c = \sqrt{\beta^2 n^2 - 1}$$

$$\approx \sqrt{2(1 - 1/n \beta)}$$

for small $\theta_c$, e.g. in gases.  

(32.43)