1. Use the latest version of the PDG (online or hard copy) when answering the following:

(a) What is the branching ratio for a $Z$ to decay to $e^+e^-$?

(b) Is this branching ratio consistent with the branching ratios for $Z \rightarrow \mu^+\mu^-$ and $Z \rightarrow \tau^+\tau^-$ to within current measurement errors?

(c) What are the lifetimes of the neutral and charged pions, $\pi^0$ and $\pi^\pm$? (As we’ll see, the $\pi^0$ undergoes electromagnetic decay while the $\pi^\pm$ undergoes weak decay, leading to the huge difference in lifetimes.)

(d) How far on average would a $\pi^0$ with total energy 2 GeV travel before decaying? How about a $\pi^+$? Approximate answers are sufficient. The point is to notice how very different the detection techniques might need to be when dealing with these otherwise closely related particles.

2. Griffiths 2.2 (light-by-light scattering)

3. Griffiths 2.3 (Compton scattering)

4. Relativity review. High-energy protons traveling through intergalactic space are passing through a sea of cosmic microwave background photons, whose average black-body temperature is 2.7 K. For a sufficiently energetic proton the reaction $p + \gamma \rightarrow p + \pi^0$ can take place, where the photon ($\gamma$) here is a cosmic microwave photon. Calculate the minimum proton energy $E_p$ required for this reaction to take place. To do this, use the fact that the relativistic invariant $m$ (i.e., the invariant mass of the system) must be the same in the lab frame and the center of mass frame. Since we’re looking at the threshold case, the final-state particles are at rest in the center-of-mass frame. Write your initial equation(s) without approximation, but then introduce whatever approximations you like on your way to an expression for $E_p$ and an order-of-magnitude numerical answer. The algebra needn’t be grungy here.

Some side notes: At energies higher than $E_p$, the proton mean free path through space is greatly reduced. This so-called Greisen-Zatsepin-Kuzmin (GZK) limit is observed as a dramatic drop-off at high energy in the spectrum of cosmic ray protons. The threshold calculation in this homework problem is easily found online and elsewhere – don’t go looking for it! However, you should skim the “GZK limit” wikipedia page to check your numerical answer. Unless you do a much more complicated calculation than this problem requires, you will get an answer higher than the standard GZK limit of $5 \times 10^{19}$ eV. Why might your answer be higher?

5. Griffiths 6.6 (makeshift $\pi^0 \rightarrow \gamma\gamma$ lifetime calculation)

6. Griffiths 6.8 (a cross section calculation). It’s worth reading problems 6.9 and 6.10 to get a feel for other scenarios whose differential cross section you could have calculated. Problem 6.8 isn’t the most broadly applicable scenario, but the others require much more algebra for no pedagogical gain.