Gyroscopes

- basic precession
- precession
- nutation

Basic picture

Bike tire demo first

Recall \( \vec{\tau} = \frac{d\vec{L}}{dt} \) (cf. \( \vec{F} = \frac{d\vec{\dot{r}}}{dt} \))

So, in time \( dt \):

\[ \vec{L} = \vec{L}_0 + \vec{\tau} dt \]

\[ \vec{L} \perp \vec{L}_0 \text{ always} \]

(by construction of system)

\[ \Rightarrow |\vec{L}| = \text{const.} \]

\( \vec{L}(t) \) describes a circle:

The circular motion resulting from \( \vec{\tau} \) on \( \vec{L} \) is called "precession".
Notice, in time $dt$:

$$\frac{dl}{dt} = L \frac{d\phi}{dt}$$

or

$$\frac{dl}{dt} = L \frac{d\phi}{dt}$$

rate of precession

$$\dot{\Omega} = \frac{d\phi}{dt}$$

($\Omega$ = omega)

$$\Rightarrow \dot{\Omega} = \frac{\dot{\phi}}{L}$$

and with vectors

$$\dot{r} = \hat{\Omega} \times \dot{r}$$

"$\sin \theta$" part of $\dot{r}$ works:

Here, $dL = d\phi \cdot L \sin \theta$  

radius of circle made by tip of $L$

$$\Rightarrow \frac{dL}{dt} = \dot{\phi} L \sin \theta$$

$$\dot{r} = \hat{\Omega} \times \dot{r}$$

works.

Compare with circular centripetal motion

$\vec{F}$ by construction & $\vec{F}$ fixed

$\Rightarrow$ circular motion

if $\dot{\phi} = 0$, mass "falls" inward due to $\vec{F}$, but that's a special case.

$\vec{F} = \frac{d\vec{r}}{dt}$  

so resulting motion depends on what $\vec{F}$ is as well as $\vec{F}$. 
with $\vec{c}$, $\vec{z}$:

$\vec{c} \perp \vec{z}$ always; $\vec{c}$ describes a circle.

If $L_z = 0$, system "falls" in the "intuitive" way, but that, too, is a special case. $\vec{c} = \frac{dL_z}{dt}$, so motion depends on $L_z$ and $\vec{c}$.

Energy & $L_z$ conservation

precessional motion requires $KE = \frac{1}{2} I_{p\theta} \Omega^2$

$\rightarrow KE$ comes from potential released upon falling a little bit:

$\rightarrow L_z \rightarrow L \rightarrow \vec{L}$

$mgh$ provides energy for precession.

Also:

\[\begin{align*}
\text{before} & \quad \rightarrow L_z \\
\text{after} & \quad \rightarrow L_z
\end{align*}\]

If $L_y$ went from 0 to not 0, a torque would have been required, but $\vec{c}$ a torque in $\vec{z}$ direction.

The precessional motion, though, also counts in $L_z$, and

$L_y + I_{p\theta} \Omega = 0 \quad \rightarrow$ precessional angular momenting exactly balances the "extra" vertical angular momentum from the force
Can use forces & torque to understand gyro systems, or can use E/L arguments. The latter is often easier.

**Demos**

- top
- Earth as gyro:
  - fancy gyro

\[ |F_1| > |F_2| \Rightarrow \text{torque} \Rightarrow \text{precession} \quad (\Omega = \frac{2\pi}{26,000 \text{yr}}) \]

**Nutrition**

- stable height, smooth precession
- But if you "drop" it, will overshoot the equilibrium height \( \Rightarrow \text{oscillation about equilibrium height} \) (not S.H.O. !)

Overshoots \( \Rightarrow \Omega \text{ goes up to balance by from wheel.}
\text{On way back up (due to insufficient torque to maintain \( \Omega \),) returns to starting height: } \Omega = 0 \text{ briefly.} \)
Can superimpose an \( \vec{\Omega}_0 \) on the system

\[ \text{or} \quad \vec{\Omega}_0 = 0 \]

\( \vec{\Omega}_0 \) in the same direction as \( \vec{J}_\text{precess} \)

\( \vec{\Omega}_0 \) opposite to \( \vec{J}_\text{precess} \)

Aside: Free rotation of complex objects

"torque free" precession

Demo: Spinning things in the air