Newton’s Laws

Talked last time about \( \ddot{a} \); now how to cause \( \ddot{a} \).

[1] Inertia: an object stays at rest or in uniform motion in a straight line if no force acts on it. “Demo” \( \text{[Book]} \). [1]

\[
\begin{align*}
\vec{F} &= m\ddot{a} \\
F_x &= m\ddot{a}_x \\
F_y &= m\ddot{a}_y \\
F_z &= m\ddot{a}_z
\end{align*}
\]

More generally: \( \vec{F} = \frac{d}{dt}(m\vec{V}) \) if \( m \) is constant \( \Rightarrow m \frac{d}{dt}\vec{V} = m\ddot{a}\).

[2] \( \vec{F}_2 = -\vec{F}_1 \): every action (“force”) has an equal and opposite reaction. “Demo: force probes on cars on track.”

Statics

If an object has \( \ddot{a} = 0 \), \( \vec{F} \) must be zero since \( \vec{F} = m\ddot{a} \).

→ Use this fact to answer questions about the forces present.
Note: really $\sum F_i = ma$, so:

\[ \sum \vec{F_i} = -\vec{F_1} \]

\[ \sum \vec{F_i} = 0 \]

\[ \vec{F_{net}} = 0 \]

or just...

\[ \vec{F} = 0 \]

Example

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\text{tension in rope: each bit of rope pulling on neighboring bits}
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If $\vec{a} = 0$, then $\sum \vec{F_i} = 0$, so:

\[ 0 = \vec{F_x} \]

\[ = \sum [T_1 \sin \theta_1 - T_2 \sin \theta_2] + \sum [T_1 \cos \theta_1 + T_2 \cos \theta_2 - mg] \]

If $(\text{vector expression}) = 0$, then each component is zero:

\[ \begin{cases} 
T_1 \sin \theta_1 - T_2 \sin \theta_2 = 0 \\
T_1 \cos \theta_1 + T_2 \cos \theta_2 - mg = 0 
\end{cases} \]

Can solve for $T_1$ \& $T_2$, if know $mg, \theta_1, \theta_2$.

\[ T_2 = mg \frac{\sin \theta_1}{\sin(\theta_1 + \theta_2)} \]
Could have done:

- Tension $T_1$ (same piece of rope)
  (Cordal pulley)

Gives additional equation from "diagram"

\[
\begin{align*}
&T_1 \uparrow \\
&m_2 \downarrow \\
-mg \uparrow
\end{align*}
\]

\[\Rightarrow T_1 - mg = 0\] (if static)

More 3rd law demo:
- Bowling ball "rocket"
- Rocket!

Torque

\[\sum F_i = 0 \Rightarrow \vec{\alpha} = 0\]

Two ways things can move:
- Translation: $\vec{a}$, $\vec{v}$, $\vec{r}$ ($x$, $y$)
- Rotation: $\vec{\alpha}$, $\vec{\omega}$, $\vec{\theta}$

For now: $\vec{\alpha} = \vec{\omega} = 0$ (statics)

So $\vec{\alpha} = 0$

\[\vec{F} = m\vec{a}\]

2nd law

\[\vec{\tau} = I \vec{\alpha}\]

Will come back to $\vec{a}$, $\vec{v}$, $I$, etc.
Later in the term.
Define torque: \( \vec{T} = \vec{r} \times \vec{F} \)

- Here, \( \vec{r} \perp \vec{F} \) so \( \vec{T} = r \vec{F} \)

Where \( \vec{T} = \vec{r} \times \vec{F} \rightarrow \vec{T} = r \vec{F} \sin \theta \)

The direction of \( \vec{T} \) is by right-hand rule (into page)

Typically, you don't need the vector expression when working in a plane like this.

\[ \vec{T} = r \vec{F} \sin \theta \]

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Note: in static situations, can define/calculate \( \sum \vec{T} \) around any pivot point you want. Proof:

\[ \vec{T} = \sum_{i} \vec{r}_{i} \times \vec{F}_{i} \]

Shift pivot by \( \delta \):

\[ \sum_{i} (\vec{r}_{i} + \delta) \times \vec{F}_{i} \]

Same torque:

\[ \sum_{i} \vec{r}_{i} \times \vec{F}_{i} \]

\[ \sum_{i} \vec{r}_{i} \times \vec{F}_{i} \]

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Example

Q: What is \( T \)?

In general, could write 3 equations:

\[
\sum F_x = 0 \\
\sum F_y = 0 \\
\sum \tau = 0
\]

But if we put our torque pivot point at the "hinge" of the bridge, \( \tau \) falls out of \( \sum \tau = 0 \) since \(\tau = \vec{r} \times \vec{F} = 0 \).

So,

\[
\sum \tau = 0 \Rightarrow -Mg \frac{L}{2} \cos \theta + TL \sin \alpha = 0
\]

or \( \sin (90^\circ + \theta) \)

\[
\Rightarrow \alpha = 90^\circ - \left(90^\circ - \theta\right)
\]

from sketching various triangles

If we picked a different pivot for our \( \tau \) calculation, we would need all three equations to solve for \( T \). That's okay, just more work.

\[
\Rightarrow T = \frac{Mg \cos \theta}{2 \cos \left(90^\circ - \theta\right)}
\]